Erasure, Capture, and Random Power Level Selection in Multiple-Access Systems

ISRAEL CIDON, MEMBER, IEEE, HAREL KODESH, AND MOSHE SIDI, SENIOR MEMBER, IEEE

Abstract—**A communication system with many nodes accessing a common receiver through a time-slotted shared radio channel is considered. Ideally, each transmission of a node is heard by the receiver. In practice, however, due to topological and environmental conditions, the receiver is prone to fail to hear some or all of the packets transmitted in a slot. The phenomena of failing to detect *any* packet is called *erasure*, while detecting a *single* transmission out of many is called *capture*.**

This paper introduces multiple-access algorithms that handle erasures as well as captures. The algorithms are evaluated according to the maximal throughput that they can support for a Poisson arrival process. An example is given which shows that, in practice, the positive effect of captures compensates the negative effect of erasures.

In addition, we introduce a new approach to effectively exploit the capture phenomena. This approach incorporates a random power level selection scheme that allows each node to randomly choose to transmit in one of several allowable levels of power. Design issues such as number of levels, selection schemes, etc., are discussed.

I. Introduction

THE collision resolution algorithm (CRA) proposed by Capetanakis [1] and Tsybakov and Mikhailov [8] has been devised to enable the nodes in a slotted ALOHA type network to exploit the channel history for resolving collisions among competing nodes. Originally, in devising the algorithm, it has been assumed that each slot can be either a) *idle slot;* no packet is transmitted or b) *success slot;* exactly one packet is transmitted, or c) *collision slot;* two or more packets are transmitted and none is correctly received. It has been further assumed that the receiver is able to discriminate between idle, success, and collision slots, and transmit appropriate feedback signals, LACK, ACK, and NACK, respectively.

Ideally, when each transmission of a node is heard by the receiver and when the forward channel is noiseless, the above feedback signals are always faithful. In practice, however, due to topological and environmental conditions, the receiver is prone to fail to hear some or all of the packets transmitted in a slot. The phenomena of failing to detect *any* transmission is called *erasure*, while detecting a *single* transmission out of many is called *capture*. The reasons for erasures and captures in practical systems are that mobile users (nodes) may occasionally be hidden (for example, because of physical obstacles), or have different distances from the receiver, or transmit in different power levels, or because of fading problems. Note that whenever an erasure occurs, a false LACK (instead of ACK or NACK) is sent by the receiver, and whenever a capture occurs an ACK is sent with the identity of the node that captured the receiver. In addition, due to additive noises that are intrinsic in any physical radio channel, the receiver may detect a collision instead of an idle or a success slot. The latter is referred to as *noise error*.

Multiple-access algorithms that handle noise errors were presented in [5], [9] and in [6]. In [5], [9], the proposed algorithm is based on the tree collision resolution algorithm (CRA) [1], [8], while that in [6] is based on Gallager's 0.487 algorithm [4], [10]. Erasures have been handled in [3]. A deterministic capture model in which the nodes of the network are divided into priority groups has been studied in [2], [7].

In this paper, we propose and analyze (in Section III) the performance of tree-like algorithms that can handle erasures, captures, and noise errors. The algorithms are similar to those presented in [3]. A remarkable feature of these algorithms is that they ensure that all packets are eventually transmitted, including the erased packets, whenever the arrival rate of new packets to the system is less than the maximal throughput that the algorithms can support. An example is given which shows that, in practice, the positive effect of captures compensates the negative effect of erasures.

As opposed to [2], [7] where capture has been modeled as a deterministic phenomena, the model for capture used in this paper is probabilistic and therefore more realistic. This model motivates a new approach (introduced in Section IV) to effectively exploit the capture phenomena. This approach incorporates a random power level selection scheme that allows each node to randomly choose to transmit in one of several allowable levels of power. Design issues such as how many levels should be used, how to select the levels, etc., are discussed. We show that a throughput as high as 0.592 can be achieved in a two-power level system.

II. The Model

We consider a communication system that consists of many nodes (practically infinite number) accessing a common receiver. The forward channel is assumed to be a time-slotted radio channel. In a given slot, each node can transmit, at most, one packet whose duration is one time slot. The beginning of a transmission in synchronized with the beginning of a time-slot. Due to topological and environmental conditions, a packet might or might not be heard by the common receiver.

During any time-slot one of the following events may occur: a) idle slot—either none of the nodes of the network is transmitting or several nodes are transmitting but none of them is heard by the receiver (erasure). For an idle slot, the receiver sends a LACK feedback signal that is received by all nodes of the network. b) Success slot—either a single node is transmitting and being received properly or one node out of several transmitting nodes is being properly received by the receiver (capture). For a success slot, the receiver sends an ACK(i) feedback signal (i is the identity of the node whose packet is received properly) to all nodes. c) Collision slot—either a number of nodes are transmitting, and at least two of them are

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I. Cidon was with the Department of Electrical Engineering, Technion—Israel Institute of Technology, Haifa, 32000 Israel. He is now with IBM Thomas J. Watson Research Center, Yorktown Heights, NY 10598.

H. Kodesh was with the Department of Electrical Engineering, Technion—Israel Institute of Technology, Haifa, 32000 Israel. He is now with Aitech, Herzelia, Israel.

M. Sidi is with the Department of Electrical Engineering, Technion—Israel Institute of Technology, Haifa, 32000 Israel.

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heard by the receiver, or no node, or a single node is transmitting but the receiver interprets it as a collision (noise error). For a collision slot the receiver sends a NACK feedback signal to all nodes.

The model for captures, erasures, and noise errors that we use is probabilistic. Specifically, when $n \geq 1$ nodes transmit, the probability of erasure is $\pi_{n,0}$ and when $n \geq 2$ nodes transmit, the probability of capture is $\pi_{n,1}$. Also, the respective probabilities of interpreting an idle or a success slot as a collision are $\pi_{0,C}$ and $\pi_{1,C}$. We assume that erasures, captures, and noise errors are probabilistically independent and that $\pi_{n,0} + \pi_{n,1} < 1$ $n \geq 2$.

One should observe that with this model, although the receiver broadcasts the same information to all nodes, it may happen that different nodes will have different knowledge about what really happened during a particular slot. To see that, assume that some nodes have transmitted a packet in a certain slot and were acknowledged by a LACK. Obviously, these nodes are aware of the error made by the receiver; however, no other node in the system is. Likewise, when some node i transmits a packet in a certain slot and is acknowledged by an ACK(j) ($i \neq j$), it is aware of the error made by the receiver. Subsequently, those nodes whose packets were not heard by the receiver will be considered to belong to a *lapsed set* until they retransmit their packets again.

III. Tree Algorithms

In this section, we introduce tree-based multiple-access algorithms similar to those of [3] for channels with captures, erasures, and noise errors. If the channel were free of any kind of error then the tree CRA is as follows [5]. After a collision, all nodes involved flip a binary coin; those flipping 0 retransmit in the very next slot; those flipping 1 retransmit immediately after the collision (if any) among those flipping 0 has been resolved; no new packets may be transmitted until after the initial collision is resolved. It is said that a conflict is resolved precisely when all nodes of the system become aware that all initially colliding packets have been successfully retransmitted. The time elapsed from an initial conflict until it is resolved is called a conflict-resolution-interval (CRI).

As is well known [5], the presence of noise errors does not require any changes in the tree CRA. However, if erasures and captures occur, one should determine the actions taken by nodes that join the lapsed set because their packets were not heard by the receiver. We consider two schemes.

a) The *Wait Scheme:* All nodes that transmit in a given slot and are not heard by the receiver either due to an erasure or due to a capture, retransmit at the beginning of the next CRI;

b) The *Persist Scheme:* All nodes that transmit in a given slot and are not heard by the receiver, retransmit in the subsequent slot.

Recall that node i learns about its failure to be heard by the receiver, by examining the feedback indication; if it transmits and the feedback indication is a LACK or ACK(j) with $j \neq i$—then node i knows that it has not been heard by the receiver and therefore it joins the lapsed set.

Regarding the first-time transmission rule, namely, which packets are transmitted for the first time at the beginning of a CRI, we adopt the idea proposed in [4], [5] to "decouple" the transmission times from arrival times. We define an *arrival epoch* of length τ where the ith arrival epoch is the semi-opened interval $(i\tau, (i+1)\tau]$. The rule that is used is to transmit a new packet that arrived during the ith arrival epoch in the first utilizable slot following the CRI for new packets that arrived during the $i-1$ arrival epoch [5]. Here τ is a fixed length epoch adjusted to maximize the achievable throughput.

Note that in addition to packets that are transmitted for the first time at the beginning of a CRI (according to the above rule) some *residual* packets are also transmitted. For the Persist scheme and the Wait scheme, the residual packets are those packets that join the lapsed set during the *last* slot and during *all* slots of the previous CRI, respectively.

A. Analysis of the Wait and the Persist Schemes

In this section, we analyze the performance of the Wait and the Persist schemes. Since our main goal is to study the mutual influence between captures and erasures, and since the incorporation of noise errors into the analysis is very simple [3], we assume here that $\pi_{0,C} = \pi_{1,C} = 0$. Our goal in this section is to determine the maximal output rate (throughput) attainable with the Wait and the Persist schemes.

In a fashion similar to [3], we introduce the following definitions: X_i is the set of *all* packets transmitted during the first time slot of the ith CRI; A_i is the set of *new* packets transmitted in the first time slot of the ith CRI; Y_i is a set containing the packets that are residual at the end of the ith CRI. These packets are referred to as the ith *residual packets* and are retransmitted at the beginning of the $(i+1)$st CRI. Obviously,

$$X_i = A_i + Y_{i-1}. \tag{1}$$

Assuming that $\{A_i, i \geq 1\}$ is a sequence of i.i.d. random variables, it is clear that given Y_{i-1}, the random variable Y_i is independent of Y_j for $j < i-1$. Consequently, $\{Y_i, i \geq 0\}$ forms a Markov chain. To proceed, we first need to determine the transition probabilities of this chain:

$$p(n_2/n_1) = \text{Prob}\{Y_i = n_2/Y_{i-1} = n_1\}. \tag{2}$$

To that end, let $P_n(l)$ be the probability for l residual packets at the end of a CRI that started with n packets in its first time slot, namely $P_n(l) = \text{Prob}\{Y_i = l/X_i = n\}$. Let $P_A(m) = \text{Prob}\{A_i = m\}$. Then

$$p(n_2/n_1) = \sum_{m=0}^{\infty} P_{n_1+m}(n_2) P_A(m). \tag{3}$$

Let $Q_i(n)$ be the probability that i nodes out of n nodes will flip 0; i.e., $Q_i(n) = \binom{n}{i} p^i (1-p)^{n-i}$ where p is the probability that a node will flip 0. Then we can calculate $P_n(l)$ as follows [see explanation following (5d)]:

Remark: In the following equations, $P_n(l) \equiv 0$ for $l < 0$ and for $l > n$.

For the Wait scheme:

$$P_0(0) = 1;\ P_1(0) = 1 - \pi_{1,0};\ P_1(1) = \pi_{1,0} \tag{4a}$$

$$P_n(l) = (1 - \pi_{n,0} - \pi_{n,1}) \sum_{i=0}^{n} Q_i(n) \cdot \sum_{k=0}^{l} P_i(k) P_{n-i}(l-k) \quad n \geq 2;\ 0 \leq l \leq n-2 \tag{4b}$$

$$P_n(n-1) = \pi_{n,1} + (1 - \pi_{n,0} - \pi_{n,1}) \sum_{i=0}^{n} Q_i(n) \{P_i(i-1) \cdot P_{n-i}(n-i) + P_i(i) P_{n-i}(n-1-i)\} \quad n \geq 2 \tag{4c}$$

$$P_n(n) = \pi_{n,0} + (1 - \pi_{n,0} - \pi_{n,1}) \sum_{i=0}^{n} Q_i(n) P_i(i) \cdot P_{n-i}(n-i) \quad n \geq 2; \tag{4d}$$

For the Persist scheme:

$$P_0(0)=1;\ P_1(0)=1-\pi_{1,0};\ P_1(1)=\pi_{1,0} \tag{5a}$$

$$P_n(l)=(1-\pi_{n,0}-\pi_{n,1})\sum_{i=0}^{n} Q_i(n) \cdot \sum_{k=0}^{l} P_i(k)P_{n-i+k}(l) \quad 0\le l\le n-2;\ n\ge 2 \tag{5b}$$

$$P_n(n-1)=\pi_{n,1}+(1-\pi_{n,0}-\pi_{n,1})\sum_{i=0}^{n} Q_i(n)\{P_i(i-1) \cdot P_{n-1}(n-1)+P_i(i)P_n(n-1)\} \quad n\ge 2 \tag{5c}$$

$$P_n(n)=\pi_{n,0}+(1-\pi_{n,0}-\pi_{n,1})\sum_{i=0}^{n} Q_i(n)P_i(i)P_n(n) \quad n\ge 2. \tag{5d}$$

The explanation of (4) and (5) is simple. For instance, (4b) expresses the fact that (for the Wait scheme) a CRI that started with n transmitted packets ends with l residual packets $0 \le l \le n - 2$ if: 1) the n packets did not join the lapsed set in the first slot of the CRI; and 2) after the nodes were split into i nodes and $n - i$ nodes according to the distribution $Q_i(n)$, the sum of residual packets from the set of i nodes and the set of $n - i$ nodes should equal l. The reasons for (4c) and (4d) are similar to that of (4b) except that now a single packet out of n packets might be captured (with probability $\pi_{n,1}$) or the n packets can be erased (with probability $\pi_{n,0}$) in the first slot of the CRI, respectively. The explanation of (5b) is similar. Here a CRI starts with $n \ge 2$ packets and ends with $0 \le l \le n - 2$ residual packets. Therefore, the first slot of the CRI must be a conflict slot. Then the nodes are split into sets of i and $n - i$ nodes. When the CRI of the i nodes ends, any number $0 \le k \le i$ of nodes may belong to the lapsed set. Consequently, according to the Persist scheme, the second CRI starts with $n - i + k$ packets, from which exactly l should belong to the lapsed set at the end. Equations (5c)–(5d) are explained in a similar manner.

From (4) and (5) the probabilities $P_n(l)$ $n = 0, 1, 2, \cdots$ $0 \le l \le n$ can be computed recursively. Therefore, using (3), the steady-state probabilities ($P_Y(k)$ $k \ge 0$) of the chain $\{Y_i,\ i \ge 0\}$ can be computed (assuming that it is ergodic) via

$$P_Y(k)=\sum_{j=0}^{\infty} P_Y(j)p(k/j) \quad k\ge 0; \qquad \sum_{k=0}^{\infty} P_Y(k)=1. \tag{6}$$

The following theorems (whose proofs appear in Appendix A) state conditions upon $\pi_n = \pi_{n,0} + \pi_{n,1}$ that are sufficient for the Markov chain $\{Y_i,\ i \ge 0\}$ to be ergodic in the Wait scheme and in the Persist scheme. Note that these conditions are much more general than the corresponding conditions stated in [3].

Theorem 1: Let a system operate with the Persist scheme. Let $\pi_n = \pi_{n,0} + \pi_{n,1}$. If $\pi_n < 1$ $\forall n \ge 1$ and $\exists M > 1$, $0 \le \eta < 1$ such that $\forall n \ge M$ holds $\pi_n/p(1 - \pi_n) \le \eta$, then the Markov chain $\{Y_i,\ i \ge 0\}$ is ergodic (recall that p is the probability that a node flips 0 in the resolution algorithm).

Theorem 2: Let a system operate with the Wait scheme. Let $\pi_n = \pi_{n,0} + \pi_{n,1}$, $a(n) = Q_0(n) + Q_n(n)$ and $b(n) = \sqrt{1 + \sqrt{n - 1}/n}$. If $\pi_n < 1$ $\forall n \ge 1$ and $\exists M > 1$ such that $\forall n \ge M$ holds $\pi_n \le (1 - a(n))(1 - b(n))/[a(n) + b(n)(1 - a(n))]$, then the Markov chain $\{Y_i,\ i \ge 0\}$ is ergodic.

In less formal words, Theorems 1 and 2 say that for the Persist scheme Y_i is ergodic if for large n, $\pi_n < p/(1 + p)$, and for the Wait scheme Y_i is ergodic if for large n, $\pi_n < 1/2\sqrt{n}$.

To continue with the analysis, let the average length (in slots) of a CRI that started with m new packets and k residual packets be denoted by L_n where $n = m + k$. Then

For the Wait scheme:

$$L_0=L_1=1; \tag{7a}$$

$$L_n=1+(1-\pi_{n,0}-\pi_{n,1})\sum_{i=0}^{n} Q_i(n)(L_i+L_{n-i}) \quad n\ge 2. \tag{7b}$$

For the Persist scheme:

$$L_0=L_1=1; \tag{8a}$$

$$L_n=1+(1-\pi_{n,0}-\pi_{n,1})\sum_{i=0}^{n} Q_i(n) \cdot \left[L_i+\sum_{j=0}^{i} P_i(j)L_{n-i+j}\right] \quad n\ge 2. \tag{8b}$$

The reason for (7b) is that in case of an erasure or a capture, the CRI reduces to a single idle or successful slot, respectively. If a collision occurs, the n nodes are split into i and $n - i$ nodes according to the distribution $Q_i(n)$. Consequently, we would have two sub CRI's with average lengths L_i and L_{n-i}. The reason for (8b) is similar except that in the Persist scheme, with probability $P_i(j)$ $(0 \le j \le i)$ the second sub CRI will start with $n - i + j$ packets.

From (7) and (8), the quantities L_n $n \ge 0$ can be computed recursively. Assuming that the chain $\{Y_i,\ i \ge 0\}$ is ergodic, the maximum output rate (throughput) possible with the algorithm is given by

$$T=\frac{E[A]}{\sum_{m=0}^{\infty}\sum_{k=0}^{\infty} L_{m+k}P_A(m)P_Y(k)} \tag{9}$$

where $E[A] = \sum_{m=0}^{\infty} mP_A(m)$.

In the following we will assume that packets arrive to the system according to a Poisson process with rate λ (packets/slot). Each time a CRI is started, a new epoch of length τ (in slots) is chosen, so that $P_A(m) = (\lambda\tau)^m e^{-\lambda\tau}/m!$ $m \ge 0$.

If $\lambda < T$ then the system would be stable. We notice that the attainable throughput T depends on both the epoch length τ and on the coin flipping probability p. These two parameters can be optimized so that the attainable throughput would be maximized. In the following, we will use $p = 0.5$, and in each case we maximize T over the parameter $x = \lambda\tau$. Let x^* be the optimal parameter namely the optimal average number of new packets that are transmitted (for the first time) at the beginning of a CRI, and let T^* be the maximal attainable throughput. Then for each $\lambda < T^*$, $\tau^* = x^*/\lambda$ is used.

B. Numerical Results

In this section, we describe the results that correspond to the following three cases:

1) $\pi_{n,0}=\pi$ $n=1, 2, 3$; $\pi_{n,0}=0$ $n>3$; $\pi_{n,1}=0$ $n>2$;

2) $\pi_{n,0}=0$ $n\ge 1$; $\pi_{n,1}=\pi$ $n=2, 3$; $\pi_{n,1}=0$ $n>3$;

3) $\pi_{n,0}=\pi_{n,1}=\pi$ $n=2, 3$; $\pi_{n,0}=\pi_{n,1}=0$ $n>3$; $\pi_{1,0}=\pi$.

Case 1) corresponds to partial erasure without capture, Case

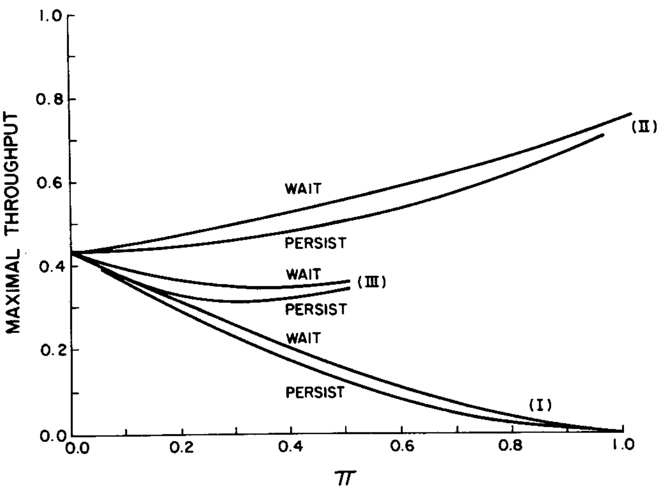

Fig. 1.

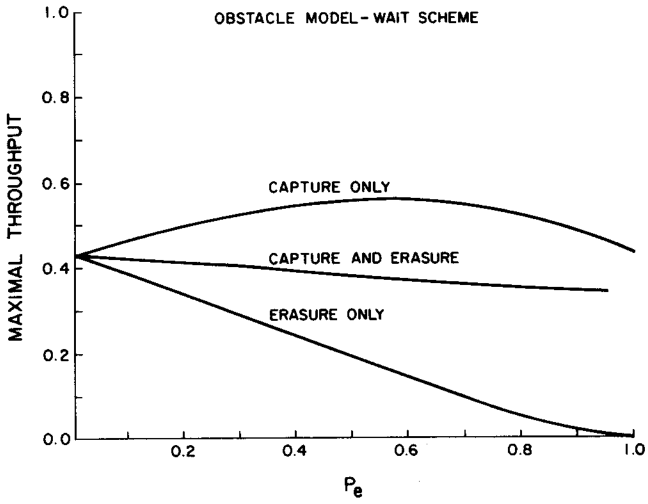

Fig. 2.

2) to partial capture without erasure, and Case 3) to partial capture and erasure (note that by Theorems 1 and 2, the Markov chain $\{Y_i, i \geq 0\}$ is ergodic in all these cases). The results for the Wait and Persist schemes for these cases are depicted in Fig. 1 (we use $p = 0.5$ as the probability for 0). In all cases, we observe that the Wait scheme is better than the Persist scheme. Consider Case 1) first. The reason that the Wait scheme is better is that with this scheme, those packets that are erased during a CRI are accumulated and sent at the beginning of the next CRI, thus increasing the number of packets transmitted at the beginning of a CRI, and thus reducing the erasure probability at the beginning of a CRI. With the Persist scheme, on the other hand, the erased packets are retransmitted again immediately when erased, thus increasing the number of collisions and erasures in a CRI. In Case 2), we see that the differences between the two schemes are smaller (percentage-wise). The reason is that reducing the probability of a capture at the beginning of a CRI (this is what happens with the Wait scheme) is bad since capture has positive effects on the attainable throughput. Still the fact that the average length of a CRI is increased (in the Persist scheme) is dominant and therefore the Wait scheme is better. For Case 3) the explanations are similar.

Without erasures and captures, it is known that the optimal average number of new packets transmitted at the beginning of a CRI is $x^* = 1.15$. In all the above cases, x^* decreases as π increases. However, in Case 1), x^* decreases to very small values when π is large, while in Case 2) the decrease is very moderate so that x^* remains between 1.15 and 1. Finally, we note that when both erasures and captures are present [Case 3)], they compensate each other. This phenomena will be observed in the next example, too.

C. The Obstacle Model

The numerical results presented above were for a rather artificial set of erasure and capture probabilities. In this section, we present a model in which the capture and erasure probabilities are more realistic, and see how the Persist and the Wait schemes perform in practical situations.

As before, all nodes access a common receiver. We assume that the nodes are mobile and are near an obstacle (e.g., a wall, high mountain, or deep valley) which prevents the receiver from hearing the transmissions of nodes that are behind it.

When no nodes are behind the obstacle, then each feedback signal transmitted by the receiver is correct. However, if all transmitting nodes are "invisible" to the receiver, then it will interpret the slot as an idle slot, even if it is actually a success or a collision slot. If all transmitting nodes, except one, are behind the obstacle, then only that single node will be received correctly, i.e., it will capture the channel. If two or more transmitting nodes are visible to the receiver, then a collision slot is observed.

Let p_e be the probability of a transmitting node being behind the obstacle and thus failing to be heard by the receiver. The capture and erasure probabilities are therefore as follows:

$$\pi_{n,0}=p_e^n \qquad n\geq 1 \tag{10a}$$

$$\pi_{n,1}=np_e^{n-1}(1-p_e) \quad n\geq 2. \tag{10b}$$

Using these values for $\pi_{n,0}$ and $\pi_{n,}$, the analysis presented in Section III-A can be applied. Note that in the obstacle model, both the erasure and the capture probabilities decrease exponentially in the number of transmitting nodes, so by Theorems 1 and 2, the Markov chain $\{Y_i, i \geq 0\}$ is ergodic in this case.

We will examine the following three cases: 1) the "capture only," in which $\pi_{n,0} = 0\ n \geq 1$; $\pi_{n,1} = np_e^{n-1}(1 - p_e)\ n \geq 2$; 2) the "erasure only," in which $\pi_{n,0} = p_e^n\ n \geq 1$; $\pi_{n,1} = 0\ n \geq 2$; 3) the "capture and erasure" (the practical case), in which (10) holds. The performance of the system in these three cases is depicted in Figs. 2 and 3 for the Wait and Persist schemes, respectively.

Case 1): The interesting phenomenon here is the peak in the graphs. The reason for this peak is as follows. The most significant contribution (due to captures) to the throughput belongs to captures happening in slots with small number of transmissions, since $\pi_{n,1}$ has relatively high values then. It is easy to see that $\pi_{n,1}$ reaches a maximum when $p_e = (n - 1)/n$. Therefore, for $n = 2\ p_e = 0.5$ and for $n = 3\ p_e = 0.667$. Consequently, the peak occurs between these values of p_e.

Case 2): The behavior here is similar to that depicted in Fig. 1.

Case 3): With both captures and erasures, we observe an interesting behavior. The channel throughput decreases as the single node "invisibility" probability increases—but this is happening in a very moderate fashion! We conclude that in practical systems the positive effect of captures compensates the negative effect of erasures in such a way that the channel is immunized against drastic changes in its maximal attainable throughput!

We observe from the graphs that the maximal attainable throughput decreases almost linearly in the Wait scheme, while it decreases for low p_e values and becomes almost constant for values exceeding 0.4 in the Persist scheme; i.e., in the Persist scheme, the capture, and erasure effects balance each other almost perfectly.

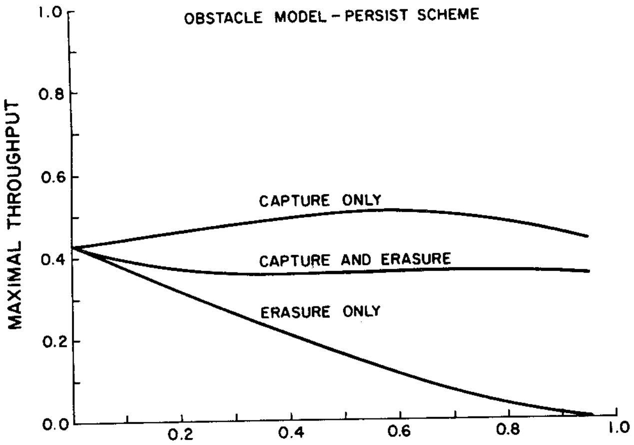

Fig. 3.

A point that deserves explanation is the fact that for p_e values beyond 0.7, the Persist scheme performs better than the Wait scheme, even though the Wait scheme is superior for either capture only or erasure only. The reason is that the Wait scheme is more sensitive to the amount of packets transmitted at the beginning of a CRI than the Persist scheme. Recall that in each point of the graphs in Figs. 2 and 3, the optimal epoch (alternatively, the optimal number of new packets transmitted at the beginning of a CRI x^*) is chosen. When both capture and erasure are present and the Wait scheme is used, the optimal epoch (for the combined effects) chosen at the beginning of a CRI causes the capture effect to contribute less to the overall throughput than in the optimal case (for capture only), and thus, despite its superiority in the capture alone and erasure alone, operating under both phenomena it becomes inferior. The Persist scheme, on the other hand, is less sensitive to the fact that different epochs are used when there is only capture, only erasure or both are present. In any case, the differences between the two schemes are minor.

IV. Random Power Level Selection

Obviously, the capture effect is a positive phenomenon. With the obstacle model, the capture was caused by geographical and environmental conditions which could not be controlled. An attempt to control the capture phenomenon has been suggested in [2], [7] using a model that consists of dominant and nondominant nodes and assuming that only dominant nodes can capture the channel. From the performance aspect, capture does contribute to the throughput, but the model lacks the "fairness" element because only the group of dominant nodes is given the privilege of capturing the channel.

In this section, we introduce and analyze a new method to exploit the capture effect in order to increase the system efficiency by proposing a new random power level selection scheme that ensures that every node in the system will be able to capture the channel with a positive probability. Moreover, since all nodes of the system perform the same scheme, it is fair in the sense that all nodes have the same chances to capture the channel.

We assume a radio network with nodes that are capable of transmitting in one of l power levels $L_1 < L_2 < \cdots < L_l$ at any slot. With the random power level selection scheme, when it should transmit (according to the multiple-access algorithm that is used), a node (randomly) selects to transmit in one of the l allowable power levels. We denote by P_k $1 \le k \le l$ the probability that a node will select level k for transmission. A node i captures the channel if its transmission power is C ($C > 1$) times greater than the sum of the power of all other transmitting nodes (C is some constant), namely if

$$L_i \ge C \sum_{\substack{j\in N\\ j\ne i}} L_j. \tag{11}$$

(N is the set of all nodes that transmit in a given slot). Note that (11) assumes that all nodes have equal distances from the receiver. For different distances the condition for capture of the channel by node i will be

$$\frac{L_i}{R_i^2} \ge C \sum_{\substack{j\in N\\ j\ne i}} \frac{L_j}{R_j^2} \tag{12}$$

where R_i is the distance of node i from the receiver. There is no essential difference between the two conditions (11) and (12) because in the latter case, an "effective power" $\tilde{L}_i$ can be defined, with $\tilde{L}_i = L_i/R_i^2$, and thus, (12) can be simplified to (11). Therefore, we will use (11) as the condition for capture. The constant C will be referred to as *Capture Factor*.

When the random power level selection scheme is applied, one can use either the Wait scheme, in which the lapsed set members are waiting for the first slot of the next CRI in order to retransmit, or the Persist scheme, in which the lapsed set members retransmit in the time slot immediately following the one in which they were not heard by the receiver. We shall consider here only the Wait scheme. As we are interested in understanding the behavior of the random power level selection scheme, we assume no erasures in the system, i.e., $\pi_{n,0} = 0$ $n \ge 1$. There is no difficulty in incorporating erasures into the model later.

The equations describing the system behavior are identical to those presented in Section III. We only need to specify the set of $\pi_{n,1}$. For the random power level selection scheme, we have for $n \ge 2$:

$$\pi_{n,1} = \sum_{i=2}^{l} \sum_{S_{n,i}} n \binom{n-1}{n_{i-1}} \binom{n-1-n_{i-1}}{n_{i-2}} \cdots \binom{n-1-n_{i-1}-\cdots-n_3}{n_2} P_i \prod_{j=1}^{i-1} P_j^{n_j} \tag{13a}$$

where P_i is the probability for a single node to select to transmit in power level $L_i (\sum_{i=1}^{l} P_i = 1)$, n_i is the number of nodes that select to transmit in power level L_i, l is the number of plausible levels in the system and for $i \ge 2$

$$S_{n,i} = \left\{ (n_{i-1}, n_{i-2}, \cdots, n_2, n_1) : \sum_{j=1}^{i-1} n_j = n-1; \sum_{j=1}^{i-1} n_j L_j \le L_i/C \right\}. \tag{13b}$$

The set $S_{n,i}$ in (13b) contains all the possible permutations for which a single node at level i captures the channel when n nodes are transmitting and in (13a) we sum the corresponding probabilities over all these permutations and over all levels.

When applying the random power level selection scheme, some questions arise, among them: a) What is the effect of a limited power range? b) How many levels should one assign for a given power range? c) What is the optimal probability distribution for the level selection algorithm? We address these questions below.

Let us first examine the simple model of two transmission levels L_{low} and L_{high}. We consider three cases: 1) $L_{\text{high}} \gg CL_{\text{low}}$; 2) $C = 10$; 3) $C = 5$. In Cases 2) and 3), we use L_{low}

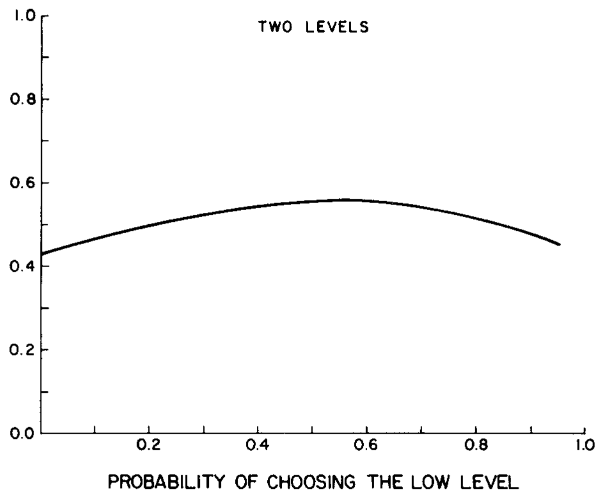

Fig. 4.

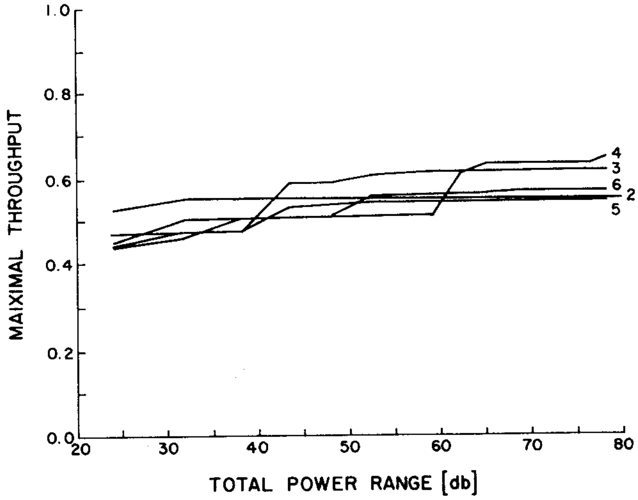

Fig. 5.

$= 1$ and $L_{\text{high}} = 5$. The results are depicted in Fig. 4 and we see that they are almost indistinguishable. Case 1) is identical to the "capture alone" case in the obstacle model because the probability for transmitting in the low power level, is playing p_e's role in this last case. In Cases 2) and 3), we have $\pi_{n,1} = 0$ $n \geq C$. Cases 2) and 3) correspond to "truncated" versions of the "capture alone" model, and the results are almost identical to Case 1). The reason is that the probability of transmission of more than three packets in a slot is fairly small. Consequently, the differences between the C's, which come into effect in (quite rare) slots containing more than five packets are negligible.

We now turn to examine a case in which several power levels are condensed in a limited predetermined power range. We assume that the random selection is uniform, i.e., $P_i = 1/l$ $1 \leq i \leq l$. We use $C = 10$ and assume that the levels are allocated in such a way that each level is R times greater than its predecessor, namely $L_i = L_{i-1}R$ $2 \leq i \leq l$. Without loss of generality, we take $L_1 = 1$. As we shall see later, this allocation of levels is very close to optimal.

Fig. 5 describes numerical results for these cases. The x axis is the total power range in decibels. $[L[\text{db}] = 20 \log_{10}(L)]$. for an l level model, the (x, y) point in Fig. 5 states that in case $R = 10^{x/(20(l-1))}$; $P_i = 1/l$, the maximal attainable throughput is y. We have examined the cases of 2–6 transmission levels and 24 to 80 dB total power range. Examining Fig. 5 reveals the following. For a given number of power levels, the maximal attainable throughput increases moderately until a point where the ratio between two levels exceeds C. At that point, since a capture becomes much more probable, we observe a drastic increase in the maximal attainable throughput. Thus, the throughput increase for three levels occurs when the three levels are spaced in such a way that the ratio between every adjacent level pairs exceeds the capture factor (10 in our case—that corresponds to 40 dB points in the graph).

Similarly, a six-level system exhibits a sharp throughput increase when the power ratio between two adjacent levels exceeds $\sqrt{10}$, namely the power ratio between level i and level $i - 2$ exceeds the capture factor. (An additional increase of the maximal throughput will occur, when the ratio between two *adjacent* levels will exceed the capture factor—this will necessitate a 100 dB power range.)

It can be concluded from the graph that in practice, when the power range does not exceed 40 dB, a two-level system will optimally exploit the capture effect. Similarly, in the range between 40–60 dB three levels are optimal and between 50–80 dB four levels are optimal. Increasing the number of levels in a given power range will yield lower maximal throughput. The reason is that too many levels cause numerous combinations that inhibit capture situations; (for instance, a pair of nodes transmitting in two adjacent levels with a ratio smaller than C). On the other hand, using too few levels will prevent exploitation of the capture effect to its full extent. Obviously, if we could expand the *total* power range, the optimal number of levels would become bigger.

We also checked the influence on the maximal attainable throughput of choosing the levels according to a nonuniform probability distribution. We observed that the distribution that yields the maximal attainable throughput is the one that degenerates an over-dense model to a model in which the spacing is such that the number of capture positions is maximal. (Thus, the six-level model, in which the ratio between every two adjacent levels is $\sqrt{10}$, degenerates to a three-level model by giving an equal probability for being in an odd level, and a zero probability for the even levels, or vice versa.) If we fix the number of levels and require that the power of level i will be C times the power of level $i - 1$, then we found that a uniform distribution is preferred.

To understand the effect of various power levels allocations within a prescribed power range, we investigated in more details the case of three (equally likely to be chosen) levels. In this case $L_1 = 1$, L_3 is the maximal allowable power and one has only to allocate L_2. Table I contains a summary of the results for two cases ($L_3 = 256$ and $L_3 = 625$) when $C = 10$. We present only L_2 levels for which significant changes occur in the throughput. As we observe, the original allocation (same ratio between adjacent levels) which is $L_2 = 16$ when $L_3 = 256$ is only 2 percent worse than the optimal allocation ($L_2 = 10.01$). When $L_3 = 625$, the difference between $L_2 = 25$ (original) and $L_2 = 20.01$ (optimal) is only 0.5 percent.

Finally, we also designed access algorithms for the random power level selection environment, that utilize the modification suggested by Massey [5] (that in absence of capture, yields throughput of 0.468). The goal was to obtain higher throughput for this environment. The analysis of the algorithms (Wait and Persist) is similar to the analysis presented in Section III. For the case of two levels, when $L_{\text{high}} \gg CL_{\text{low}}$, the Wait scheme yields the maximum throughput—0.592 (compared to 0.557 when the algorithm of Section III is used). In both cases, we optimized over the parameters of the algorithms, namely, the epoch length τ, the coin flipping probability p, and the probability of choosing one of the power levels. Note that in presence of noise errors, algorithms that are based on Massey's modification may lead to deadlocks [5]. The same can occur if erasures are present and the Wait scheme is employed. The algorithms of Section III do not require any changes if noise errors are present, and the Wait scheme can be used there even in the presence of erasures.

V. Summary

In this work, we presented tree-based collision resolution algorithms that are capable of handling captures and erasures. Two schemes, the Wait scheme and the Persist scheme, have been suggested and analyzed. From our analysis, we conclude that the Wait scheme is better.

In addition, we proposed a method, the random power level selection scheme, for initiating captures in order to increase the maximum possible utilization of the channel. In this method, a node transmits in one of several allowable transmission levels. We stated the conditions for captures and gave the rules for how to choose the number of levels, how to select the levels, etc.

Appendix

To prove Theorem 1, we need the following two Lemmas where we assume that the Persist scheme is used.

Lemma a): Assume that $\pi_{n,0} < 1\ \forall n \geq 1$. Further assume that $\exists M_1 > 1$, $0 \leq \xi < 1$ such that $\forall n \geq M_1$ holds $\pi_{n,0}/(1 - \pi_{n,0})[1 - Q_n(n) - Q_0(n)] \leq \xi$. Then $\exists \beta\ (0 \leq \beta < 1)$ such that $P_n(n) \leq \beta\ \forall n \geq 1$.

TABLE I

$L_3 = 256$		$L_3 = 625$	
L_2	Throughput	L_2	Throughput
1	0.551	10	0.551
8	0.550	11	0.607
10	0.548	19	0.606
10.01	0.604	20.01	0.617
12.8	0.599	20.8	0.616
16	0.591	25	0.614
20.01	0.599	30.05	0.615
25.6	0.581	31.23	0.610
30.01	0.529	31.47	0.600

Proof: It is easy to see that if $\pi_{n,0} < 1\ \forall n \geq 1$ then $P_n(n) < 1\ \forall n \geq 1$. Let us choose $\beta < 1$ as follows: $\beta = \max[\max_{1 \leq i \leq M_i - 1} \{P_i(i)\}, \xi]$. Then $P_n(n) \leq \beta$ for $n \leq M_1 - 1$. By induction, for $n \geq M_1$, we have

$$P_n(n) = \pi_{n,0} + (1 - \pi_{n,0} - \pi_{n,1})P_n(n)\sum_{i=0}^{n} Q_i(n)P_i(i)$$

$$\leq \pi_{n,0} + (1 - \pi_{n,0}) \Big[P_n^2(n)Q_n(n) + P_n(n)Q_0(n)$$

$$+ P_n(n) \sum_{i=1}^{n-1} Q_i(n)\beta\Big] \tag{A1}$$

$$\leq \pi_{n,0} + (1 - \pi_{n,0})P_n(n)\{Q_n(n) + Q_0(n)$$

$$+ \beta[1 - Q_n(n) - Q_0(n)]\}. \tag{A2}$$

Hence,

$$P_n(n) \leq \frac{\pi_{n,0}}{1 - (1 - \pi_{n,0})\{Q_n(n) + Q_0(n) + \beta[1 - Q_n(n) - Q_0(n)]\}}$$

$$= \beta + (1 - \beta)$$

$$\cdot \frac{\pi_{n,0} - \beta(1 - \pi_{n,0})[1 - Q_n(n) - Q_0(n)]}{\pi_{n,0} + (1 - \beta)(1 - \pi_{n,0})[1 - Q_n(n) - Q_0(n)]}$$

$$\leq \beta + (1 - \beta)$$

$$\cdot \frac{(\xi - \beta)(1 - \pi_{n,0})[1 - Q_n(n) - Q_0(n)]}{\pi_{n,0} + (1 - \beta)(1 - \pi_{n,0})[1 - Q_n(n) - Q_0(n)]} \leq \beta. \tag{A3}$$

Remarks: (A1) follows from the induction hypothesis; (A2) follows since $P_n^2(n) \leq P_n(n)$; (A3) follows from the condition upon $\pi_{n,0}$ and from the fact that $\xi \leq \beta$.

From the Lemma above it follows that:

Corollary 1): Under the assumption of Lemma a) $\exists M_2 > 1$ such that $\Sigma_{i=0}^{n} Q_i(n)P_i(i) + Q_n(n) < 1\ \forall n \geq M_2$.

Let J_n be the average number of residual packets at the end of a CRI that starts with conflict multiplicity n. We now prove the following lemma.

Lemma b): Let $\pi_n = \pi_{n,0} + \pi_{n,1}$. Assume that $\pi_n < 1\ \forall n \geq 1$. Further assume that $\exists M_3 > 1$, $0 \leq \eta < 1$ such that $\forall n \geq M_3$ holds $\pi_n/p(1 - \pi_n) \leq \eta$. Then $\exists \alpha\ (0 \leq \alpha < 1)$ such that $J_n \leq \alpha n\ \forall n \geq 1$.

Proof: It is easy to see that if $\pi_n < 1\ \forall n \geq 1$ then $J_n < n\ \forall n \geq 1$. Let us choose $\alpha < 1$ as follows: $\alpha = \max[\max_{1 \leq i \leq M_4 - 1\}} \{J_i/i\}, \eta]$ where $M_4 = \max(M_2, M_3)$. Then J_n

$\leq \alpha n$ for $n \leq M_4 - 1$. By induction, for $n \geq M_4$, we have

$$J_n = n\pi_{n,0} + (n-1)\pi_{n,1} + (1-\pi_{n,0}-\pi_{n,1}) \sum_{i=0}^{n} Q_i(n) \cdot \sum_{k=0}^{i} P_i(k) J_{n-i+k}$$

$$\leq n\pi_n + (1-\pi_n)\left[\sum_{i=0}^{n} Q_i(n)P_i(i)J_n + \sum_{i=0}^{n} Q_i(n) \cdot \sum_{k=0}^{i-1} P_i(k) J_{n-i+k}\right]$$

$$\leq n\pi_n + (1-\pi_n)\left[J_n \sum_{i=0}^{n} Q_i(n)P_i(i) + \alpha \sum_{i=0}^{n} Q_i(n) \sum_{k=0}^{i-1} P_i(k)(n-i+k)\right] \quad \text{(A4)}$$

$$= n\pi_n + (1-\pi_n)\left\{J_n \sum_{i=0}^{n} Q_i(n)P_i(i) + \alpha \sum_{i=0}^{n} Q_i(n)\{J_i - i + n[1-P_i(i)]\}\right\}$$

$$= n\pi_n + (1-\pi_n)\left[J_n \sum_{i=0}^{n} Q_i(n)P_i(i) + \alpha n(1-p) - \alpha n \sum_{i=0}^{n} Q_i(n)P_i(i) + \alpha J_n Q_n(n) + \alpha \sum_{i=0}^{n-1} Q_i(n) J_i\right]$$

$$\leq n\pi_n + (1-\pi_n)\left[J_n\left\{\alpha Q_n(n) + \sum_{i=0}^{n} Q_i(n)P_i(i)\right\} + \alpha n(1-p) - \alpha n \sum_{i=0}^{n} Q_i(n)P_i(i) + \alpha^2 \sum_{i=0}^{n-1} Q_i(n) i\right] \quad \text{(A5)}$$

$$= n\pi_n + (1-\pi_n)\left[J_n\left\{\alpha Q_n(n) + \sum_{i=0}^{n} Q_i(n)P_i(i)\right\} + \alpha n(1-p) - \alpha n \sum_{i=0}^{n} Q_i(n)P_i(i) + \alpha^2\{np - nQ_n(n)\}\right].$$

Hence,

$$J_n \leq \frac{n\pi_n + (1-\pi_n)\left[\alpha n(1-p) - \alpha n \sum_{i=0}^{n} Q_i(n)P_i(i) + \alpha^2\{np - nQ_n(n)\}\right]}{1-(1-\pi_n)\left[\alpha Q_n(n) + \sum_{i=0}^{n} Q_i(n)P_i(i)\right]} \quad \text{(A6)}$$

$$= \alpha n + n(1-\alpha)\frac{\pi_n - \alpha p(1-\pi_n)}{1-(1-\pi_n)\left[\alpha Q_n(n) + \sum_{i=0}^{n} Q_i(n)P_i(i)\right]}$$

$$\leq \alpha n + \frac{np(1-\alpha)(1-\pi_n)(\eta-\alpha)}{1-(1-\pi_n)\left[\alpha Q_n(n) + \sum_{i=0}^{n} Q_i(n)P_i(i)\right]} \leq \alpha n \quad \text{(A7)}$$

Remarks: (A4) follows from the induction hypothesis; (A5) follows by applying the induction hypothesis again; (A6) follows from the above corollary; (A7) follows from the condition upon π_n and from the fact that $\eta \leq \alpha$.

The proof of Theorem 1 now follows from the above Lemma and using Lemma 2 in the Appendix of [3].

To prove Theorem 2, we need the following lemma where we assume that the Wait scheme is used.

Lemma c): Let $\pi_n = \pi_{n,0} + \pi_{n,1}$, $a(n) = Q_0(n) + Q_n(n)$ and $b(n) = \sqrt{1 + \sqrt{n-1}/n}$. Assume that $\pi_n < 1 \ \forall n \geq 1$. Further assume that $\exists M > 1$ such that $\forall n \geq M$ holds $\pi_n \leq (1 - a(n))(1 - b(n))/[a(n) + b(n)(1 - a(n))]$. Then $\exists \alpha$ ($\alpha > 0$) such that $J_n \leq n - \alpha\sqrt{n} \ \forall n \geq 1$.

Proof: It is easy to see that if $\pi_n < 1 \ \forall n \geq 1$ then $J_n < n \ \forall n \geq 1$. Let us choose $\alpha > 0$ as follows: $\alpha = \min_{1 \leq i \leq M-1} \{(i - J_i)/\sqrt{i}\}$. Then $J_n \leq n - \alpha\sqrt{n}$ for $n \leq M - 1$. By induction, for $n \geq M$, we have

$$J_n = n\pi_{n,0} + (n-1)\pi_{n,1} + (1-\pi_{n,0}-\pi_{n,1}) \sum_{i=0}^{n} Q_i(n)[J_i + J_{n-i}]$$

$$\leq n\pi_n + (1-\pi_n)\left\{a(n)J_n + \sum_{i=1}^{n-1} Q_i(n)[J_i + J_{n-i}]\right\}$$

$$\leq n\pi_n + (1-\pi_n)\left\{a(n)J_n + \sum_{i=1}^{n-1} Q_i(n)[n - \alpha\sqrt{i} - \alpha\sqrt{n-i}]\right\} \quad \text{(A8)}$$

$$= n\pi_n + (1-\pi_n)\left\{a(n)J_n + n(1-a(n)) - \alpha \sum_{i=1}^{n-1} Q_i(n)\sqrt{n + 2\sqrt{i(n-i)}}\right\}$$

$$\leq n\pi_n + (1-\pi_n)\left\{a(n)J_n + n(1-a(n)) - \alpha\sqrt{n + 2\sqrt{n-1}}(1-a(n))\right\}. \quad \text{(A9)}$$

Hence,

$$J_n \leq n - \alpha\sqrt{n}\,\frac{(1-\pi_n)(1-a(n))b(n)}{1-(1-\pi_n)a(n)} \leq n - \alpha\sqrt{n}. \quad \text{(A10)}$$

Remarks: (A8) follows from the induction hypothesis; (A9) follows since $i(n-i)$ is minimized when $i = 1$ when $1 \leq i \leq n-1$; (A10) follows from the condition upon π_n.

The proof of Theorem 2 now follows from the above Lemma and using Lemma 3 in the Appendix of [3] (with arbitrary $\beta^* < \infty$).

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Israel Cidon (M'85) received the B.Sc. (summa cum laude) and the D.Sc. degrees from the Technion—Israel Institute of Technology, Haifa, Israel, in 1980 and 1984, respectively, both in electrical engineering.

From 1980 to 1984 he was a Teaching Assistant and a Teaching Instructor at the Technion. From 1984 to 1985 he was a faculty member with the Faculty of Electrical Engineering at the Technion. Since 1985 he has been with IBM T. J. Watson Research Center. His current research interests are in distributed algorithms and communication networks.

Harel Kodesh was born in Israel in 1958. He received the B.Sc. degree in computer engineering (cum laude) in 1980 and the M.Sc. degree in electrical engineering in 1985, both from the Technion—Israel Institute of Technology, Haifa, Israel. Currently, he is working towards the M.B.A. degree at Tel-Aviv University.

From 1980 to 1985 he was a research and development engineer in the Israel Defense Forces. Since 1985 he has been a member of the technical staff at Aitech Software Engineering Ltd., Israel. His current areas of interest are computer communication networks and concurrent programming languages.

★

Moshe Sidi (S'77–M'82–SM'87) received the B.Sc., M.Sc., and D.Sc. degrees from the Technion—Israel Institute of Technology, Haifa, Israel, in 1975, 1979, and 1982, respectively, all in electrical engineering.

From 1975 to 1981 he was a Teaching Assistant and a Teaching Instructor at the Technion in communication and data networks courses. In 1982 he joined the Faculty of the Department of Electrical Engineering at the Technion. During the academic year 1983–1984 he was a Postdoctoral Associate at the Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge, MA. During the academic year 1986–1987 he was a visiting scientist at IBM, Thomas J. Watson Research Center, Yorktown Heights, NY. His current research interests are in queueing systems and in the area of computer communication networks.