Collision Resolution Algorithms in Multistation Packet-Radio Networks

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Abstract—**The performance of a multistation packet-radio network in which the nodes of the network employ some collision resolution algorithm (CRA) for accessing a shared radio channel is analyzed. The two CRA's considered here are the binary-tree CRA (BTCRA) and the clipped binary-tree CRA (CBTCRA). The exact analysis of a multistation network with these access schemes is intractable. Therefore, we present an approximate method that captures the interactions among the nodes of different stations. The main idea of the approximation technique is to view the interferences among the nodes of different stations as independent random noises, and to compute the probabilities of these noises by taking into account the interactions between the nodes. Numerical results of the approximate analysis are presented and compared with simulations.**

I. Introduction

A new model for an hierarchical packet-radio network (PRN) has been recently introduced in [3]–[5]. The model corresponds to a packet-radio network that consists of a large number of nodes and several stations, and is called "a multistation network." In the multistation model the nodes of the network are originators of data and they transmit their data through a shared channel to the stations. The nodes are geographically distributed, possibly mobile and they have limited transmission range. The stations might be the final destinations for some packets sent by the nodes and can act as repeaters for other packets, by forwarding them to their respective destinations (other stations or nodes). The network operates as follows. A packet that is generated at some node, is forwarded to a station via the shared channel by employing some multiaccess algorithm. The station then forwards it to some other station through the backbone network of stations, and finally, the latter, transmits it on the station-to-node channel to its destination.

In [3]–[5] it has been assumed that the nodes of the network employ the slotted ALOHA protocol for accessing the shared channel. In this study we assume that the nodes employ some Collision Resolution Algorithm (CRA) [1], [6], [9]–[12]. Specifically, we analyze the performance of a multistation network in which the nodes of each subnetwork employ either the binary tree CRA (BTCRA) [1], [9], or the clipped binary tree CRA (CBTCRA) [6], [10] as the protocol for accessing the shared channel.

II. Model Description

A multistation network consists of M stations S_i, $1 \leq i \leq M$, and a large number of nodes (see Fig. 1). Each station S_i hears all

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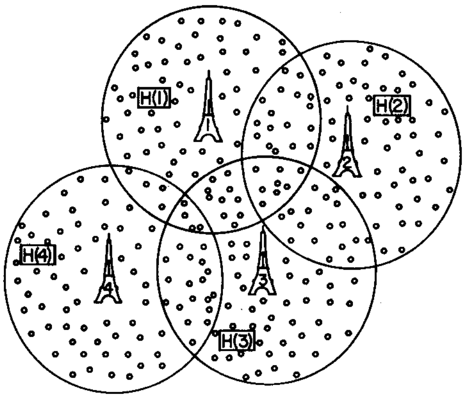


Fig. 1. A multistation network.

transmissions by the nodes that are located within some distinct (not necessarily disjoint) region R_i. Assume that the nodes of the network are clustered into M disjoint subsets, $B(i)$, $1 \leq i \leq M$. Further, assume that the nodes are mobile but nodes of $B(i)$ may move only within R_i. By definition $\cup_{i=1}^{M} B(i) = \{\text{all nodes}\}$, $B(i) \cap B(j) = \phi$ for $i \neq j$, and the transmission of a node of $B(i)$ is always heard by S_i. Station S_i together with the nodes in $B(i)$ are called *subnetwork* i. Another set of nodes associated with station S_i is the set $H(i)$ that contains all nodes that are heard by station S_i.

It is assumed that the nodes generate packetized data, such that all packets are of fixed length. The generation process of new packets by the nodes of $B(i)$ is assumed to be Poisson with mean λ_i packets per time unit (the time necessary to transmit a packet). A packet that is generated by a node of $B(i)$ is said to be forwarded, on the first time that it is correctly received by station S_i. Until being forwarded, a packet is transmitted, and retransmitted, according to the multiaccess protocol that is employed by the nodes. When forwarded, the packet leaves the multistation network, and is never retransmitted again.

All nodes of the network use a single, slotted-time, collision-type channel to forward their data packets to the stations. Time is slotted so that a packet's transmission time is exactly one time slot and all nodes are synchronized so that transmissions are within slot boundaries. A packet is correctly received at station S_i only if it is the only one transmitted by the nodes of $H(i)$.

Due to the broadcast nature of the channel, a transmitting node that belongs to $B(i)$, may be heard also by some station S_j, $j \neq i$ (in addition to being heard by S_i). In this work we assume that this event occurs with probability $\varphi_{i,j}$ and that it is independent of all other preceding and concurrent events in the network. The quantity $\varphi_{i,j}$ indicates the amount of interference of nodes in subnetwork i on nodes in subnetwork j and is called an *interference factor*.

Following every slot, feedback information is broadcasted by each station S_i through a dedicated feedback channel FC_i that is continuously monitored by the nodes of $B(i)$ (i.e., each node in $B(i)$, even those that do not have packets to transmit, monitor FC_i in each slot—this is known as full sensing). Two feedback systems are considered: binary feedback—a station S_i can detect whether there was a collision (C) on a slot [at least two simultaneous transmissions by nodes in $H(i)$], or not (NC). Ternary feedback—a station S_i can distinguish between: idle slot [no transmissions by the nodes of $H(i)$]; correct reception of a packet [a transmission by a single node of $H(i)$]; collision [transmissions by at least two nodes of $H(i)$]. Moreover, since packets carry some identification of the generating node, it is assumed that when a station S_i receives a packet correctly, it can identify whether it was sent by a node of $B(i)$ or not. Consequently, S_i will send one of the following feedback signals: NACK—for collision; ACK—for a single transmission by a node in $B(i)$; LACK—for no transmissions or a single transmission by a node not in $B(i)$.

The nodes of $B(i)$ employ a multiaccess algorithm for forwarding their packets to a station. A multiaccess algorithm is composed of: 1) first transmission rule (FTR)—determines which of the new arrivals are transmitted on every instance that the CRA allows transmissions of new arrivals (packets that were not forwarded yet), 2) collision resolution algorithm (CRA)—controls the subsequent transmissions of the nodes, for resolving the collision (i.e., getting the packets being forwarded). The time during which the a collision is being resolved is called a collision resolution interval (CRI) and the kth CRI on the ith subnetwork is denoted by $\text{CRI}_i(k)$.

Let $T_i(k)$ be the moment at which $\text{CRI}_i(k)$ ends, and let $t_i(k)$ be the latest moment for which it is known that all packets that arrived to nodes in $B(i)$ prior to time $t_i(k)$, have been already forwarded. The FTR allows all packets that had arrived to nodes in $B(i)$ during the time interval $[t_i(k), t_i(k) + \min(\Delta_i, T_i(k) - t_i(k))]$ to be transmitted at the beginning of $\text{CRI}_i(k+1)$ [6]. Δ_i is a parameter of the algorithm, and should be optimized to get the best performance of the network.

The binary-tree CRA (BTCRA) is the algorithm presented in [1], [9]. It is suitable for both binary and ternary feedback and it insures that all packets that have been transmitted at the beginning of the kth CRI, have been forwarded as the kth CRI ends. The BTCRA operates as follows. Upon a C feedback, all nodes of $B(i)$ that transmitted during the relevant slot flip a binary coin. Those flipping 0 retransmit in the following slot, while the others transmit in the slot after the collision (if any) among all 0 flippers has been resolved [9]. Instead of coin flipping one can use interval splitting [6].

An example of the operation of the BTCRA in a two-station network is presented in Fig. 2(a) along with the corresponding binary tree representation [Fig. 2(b)]. Assume that in some slot (say slot #1), in both subnetworks, a new CRI is started. In slot #1 nodes $n_1, n_2 \in B(1)$, and $n_3 \in B(2)$ transmit and $n_1 \in H(2)$. Both S_1 and S_2 hear a collision and send a C feedback. In slot #2, n_1 transmits successfully since it is the only transmitter in $H(1)$. In slot #3, $n_3 \in H(1)$, and since n_2 is transmitting also, S_1 hears a collision. S_2 hears the packet from n_3 correctly. Node n_2 transmits in slot #5, and slot #4 remains idle since none flipped 1 following slot #3. In the 2nd subnetwork, another CRI starts in slot #4 with 4 transmissions. The CRI that starts in this slot evolves with no interference from nodes in $B(1)$. In the 1st subnetwork, the CRI that starts with an idle in slot #6, is 3 slots long, since a collision was heard at S_1, due to transmissions of $n_6, n_7 \in H(1)$, and two idle slots #7 and #8 that are not interfered.

The clipped binary-tree CRA (CBTCRA) is the algorithm presented in [6], [10] without the modification by Massey [9] of saving slots that are known to contain collisions. With the CBTCRA, a new CRI is started whenever on the preceding one, there was a collision followed by exactly two successful transmissions. It might occur that not all packets that took part in the collision that started the CRI are forwarded when the CRI ends. Thus, when the new CRI starts, it may include packets that were involved in collisions during the preceding CRI, yet, their arrival distribution is the same as if they have been never transmitted [6]. Note that to implement the CBTCRA we need a ternary feedback channel. Note also that the CBTCRA can operate also under limited sensing feedback [11], i.e., a node monitors the feedback channel only when it has a packet to transmit. The latter is more reasonable in mobile environment.

If the CBTCRA would have been employed in the previous example, then the second CRI in $B(2)$ would have ended on slot #8, since there are two successful transmissions by n_6 and n_7. Following this event, n_4 and n_5, would have been retransmitted, as they would have belonged to the new arrival interval.

III. Performance Analysis

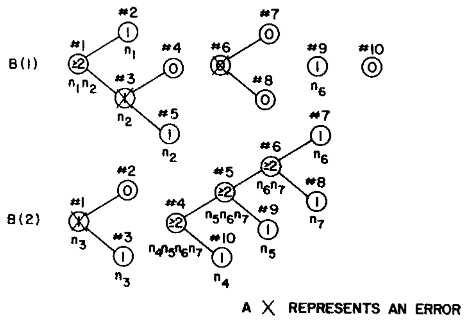
The exact analysis of the performance of a multistation network whose nodes employ a CRA is intractable since the underlying Markov chain has a very complicated structure. Consequently, we develop an approximation method from which this performance can be approximated. Due to space limitations, we present here only the analysis of the throughputs regions that the network can support. The same approach can be used to analyze the expected delay of a packet in the network. For details, the interested reader is referred to [12].

A. The Approximation Method

The idea of the approximation is to aggregate the effect of all nodes that can interfere with the transmissions of nodes in $B(i)$ into

			1	2	3	4	5	6	7	8	9	10
B(1)	transmitting nodes heard at S_2	Y	n_1									
		N	n_2	n_1	n_2		n_2				n_8	
	coin flipping following slot	0	n_1									
		1	n_2	n_2								
B(2)	transmitting nodes heard at S_1	Y			n_3	n_4		n_6n_7				n_4
		N	n_3			$n_5n_6n_7$	$n_5n_6n_7$		n_6	n_7	n_8	
	coin flipping following slot	0				$n_5n_6n_7$	n_6n_7	n_6				
		1	n_3			n_4	n_5	n_7				
S_1	transmissions heard		3	1	2	1	1	2	0	0	1	1
	feedback N/C		C	N	C	N	N	C	N	N	N	N
S_2	transmissions heard		2	0	1	4	3	2	1	1	1	1
	feedback N/C		C	N	N	C	C	C	N	N	N	N

(a)



(b)

Fig. 2. (a) An example of the BTCRA in a two-station network. (b) Binary-tree representation.

two parameters, ϵ_i and δ_i, and to determine these parameters for all subnetworks $i (i = 1, 2, \cdots, M)$, by taking into account the mutual interference among the different subnetworks.

To describe the approximation we define the following events: Whenever none of the nodes of $B(i)$ transmit at slot k, we say that event $NI_i(k)$ occurs if station S_i hears a collision in slot k. Similarly, whenever a single node of $B(i)$ transmits at slot k, we say that event $NS_I(k)$ occurs if station S_i hears a collision in slot k. These events are referred to as noise errors.

Event $NI_i(k)$ occurs only if at least two nodes of $H(i)$-$B(i)$ transmit in the corresponding slot. Event $NS_i(k)$ occurs only if at least one node of $H(i)$-$B(i)$ transmits in the corresponding slot. On both events $NI_i(k)$ and $NS_i(k)$, S_i hears a collision instead of an idle or a successful transmission, respectively, as it should have heard if it wouldn't have been exposed to the effect of the neighboring subnetworks.

The approximated analysis of the performance of a multistation network is based on the following independence assumption.

Independence Assumption: For any subnetwork i ($i = 1, 2, \cdots, M$) and for any k, the events $NI_i(k)$ and $NS_i(k)$ do not depend on the actual activity of nodes in $H(i)$-$B(i)$. Rather, whenever none of the nodes of $B(i)$ transmit, station S_i hears a collision (event $NI_i(k)$ occurs) with probability δ_i, independently of any previous and concurrent events in the network. Similarly, whenever a single node of $B(i)$ transmits, station S_i hears a collision (event $NS_i(k)$ occurs) with probability ϵ_i, independently of any previous and concurrent events in the network.

The probabilities δ_i and ϵ_i are referred to as *noise error probabilities*. Under the independence assumption, once δ_i and ϵ_i are known, the performance of subnetwork i can be evaluated as we show in Section III-C. In the sequel, the procedure for deriving δ_i and ϵ_i $(1 \leq i \leq M)$, in a manner that takes into account the mutual interference among all subnetworks, is presented.

B. Derivation of the Noise Error Probabilities

To derive the noise error probabilities δ_j and ϵ_j $(1 \leq j \leq M)$, we need the following: For the jth subnetwork $(1 \leq j \leq M)$ that operates under noise probabilities δ_j and ϵ_j we define (for $n \geq 0$):

$L_j(n, \delta_j, \epsilon_j)$—expected length of a CRI that starts with simultaneous transmission by n nodes of $B(j)$.

$L_{j,i}^{(0)}(n, \delta_j, \epsilon_j)$—expected number of slots during a CRI that starts with simultaneous transmission by n nodes of $B(j)$ in which no nodes of $B(j)$ are heard at station S_i.

$L_{j,i}^{(1)}(n, \delta_j, \epsilon_j)$—expected number of slots during a CRI that starts with simultaneous transmission by n nodes of $B(j)$ in which exactly one node of $B(j)$ is heard at station S_i.

Suppose that the values of δ_j and ϵ_j are known for all $j \neq i$. Then $L_j(n, \delta_j, \epsilon_j)$, $L_{j,i}^{(0)}(n, \delta_j, \epsilon_j)$ and $L_{j,i}^{(1)}(n, \delta_j, \epsilon_j)$ can be computed recursively [9], as is summarized in the Appendix.

Let $P_{j,i}^{(0)}$ be the probability that no nodes that belong to the set $B(j)$, are heard at S_i and let $P_{j,i}^{(1)}$ be the probability that exactly one node (out of all simultaneously transmitting ones) of $B(j)$, is heard at S_i. Let $P_j(n)$ be the probability that a CRI starts with the transmission of n nodes of $B(j)$. Following the above definitions:

$$P_{j,i}^{(l)} = \frac{\sum_{n=0}^{\infty} L_{j,i}^{(l)}(n, \delta_j, \epsilon_j) P_j(n)}{\sum_{n=0}^{\infty} L_j(n, \delta_j, \epsilon_j) P_j(n)} \qquad l = 0, 1. \tag{1}$$

The probability for no transmissions by the nodes of $H(i) \cap B(j) (j \neq i)$ is $P_{j,i}^{(0)}$. Noticing that $H(i) - B(i) = \cup_{j \neq i} H(i) \cap B(j)$, and assuming independence among the events of no transmissions by nodes of $H(i) \cap B(j)$ and no transmissions by nodes of $H(i) \cap B(l)\ \forall l \neq j$, we obtain the probability that there will be no transmissions by any of the nodes of $H(i)$-$B(i)$

$$\text{Prob}\{\text{no transmission by nodes of } H(i)\text{-}B(i)\} = \prod_{j \neq i} P_{j,i}^{(0)}. \tag{2}$$

Similarly, the probability of exactly one transmission by the nodes of $H(i)$-$B(i)$ is

$$\text{Prob}\{\text{exactly one transmission by nodes of } H(i)\text{-}B(i)\} = \sum_{l \neq i} P_{l,i}^{(1)} \prod_{j \neq i, l} P_{j,i}^{(0)}. \quad (3)$$

In a multistation network, if none of the nodes of $B(i)$ are transmitting in slot k, event $NI_i(k)$ corresponds to the event of at least two transmissions by the nodes of $H(i)$-$B(i)$ on the kth slot. Since event $NI_i(k)$ occurs with probability δ_i and the event of at least two transmissions of nodes in $H(i)$-$B(i)$ occurs with probability $1 - \prod_{j \neq i} P_{j,i}^{(0)} - \sum_{l \neq i} P_{l,i}^{(1)} \prod_{j \neq i, l} P_{j,i}^{(0)}$, we have

$$\delta_i = 1 - \prod_{j \neq i} P_{j,i}^{(0)} - \sum_{l \neq i} P_{l,i}^{(1)} \prod_{j \neq i, l} P_{j,i}^{(0)}. \quad (4)$$

Similarly,

$$\epsilon_i = 1 - \prod_{j \neq i} P_{j,i}^{(0)}. \quad (5)$$

Note that both $P_{j,i}^{(0)}$ and $P_{j,i}^{(1)}$, $j \neq i$, depend on both δ_j and ϵ_j. Consequently, (4) and (5) are actually a set of $2M$ nonlinear equations with $2M$ unknowns δ_i and ϵ_i $(1 \leq i \leq M)$ that can be solved numerically.

C. Attainable Throughputs

In this section we determine the attainable throughputs of the subnetworks. Since we are interested in determining stability regions (maximal throughputs), we assume that each time a CRI starts in subnetwork j, the epoch that is enabled is of length Δ_j (heavy traffic). Therefore, the probability of n transmissions at the beginning of a CRI that we use in (1) is

$$P_j(n) = \frac{e^{-\lambda_j \Delta_j} (\lambda_j \Delta_j)^n}{n!}. \quad (6)$$

The throughput of the jth subnetwork (TH_j) is defined to be the expected number of packets that are successfully forwarded to the backbone of stations by nodes of subnetwork j per slot. For the BTCRA, TH_j is given by

$$TH_j = \frac{\lambda_j \Delta_j}{\sum_{n=0}^{\infty} L_j(n, \delta_j, \epsilon_j) P_j(n)} \quad (7)$$

since the expected number of packets from nodes of $B(j)$ that are transmitted at the beginning of a CRI is $\lambda_j \Delta_j$ and all of them are successfully transmitted during the CRI when the BTCRA is employed.

For the CBTCRA, TH_j is given by

$$TH_j = \frac{\sum_{n=0}^{\infty} M_j(n, \delta_j, \epsilon_j) P_j(n)}{\sum_{n=0}^{\infty} L_j(n, \delta_j, \epsilon_j) P_j(n)} \quad (8)$$

where $M_j(n, \delta_j, \epsilon_j)$ is the expected number of packets from nodes of $B(j)$ that are successfully transmitted during a CRI that starts with simultaneous transmission by n nodes of $B(j)$. This quantity can be computed recursively, as we show in the Appendix.

D. Numerical Results

In this section, we present some numerical results and compare our approximation with simulations and with exact results. In the following we use the term *symmetric* two-station network whenever the network has two stations, $\varphi_{1,2} = \varphi_{2,1} = \varphi$ and the throughputs in both subnetworks are the same.

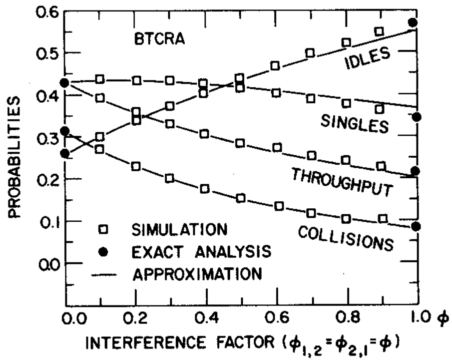


Fig. 3. Approximation and simulation: BTCRA.

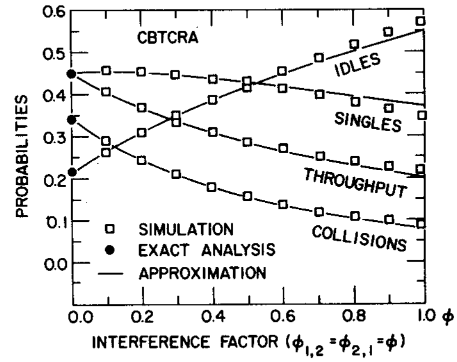


Fig. 4. Approximation and simulation: CBTCRA.

In Fig. 3, the results for the BTCRA in a symmetric two-station network are depicted. In addition to the throughput curve (as a function of φ), we included in Fig. 3 the curves for the probabilities that (in one of the subnetworks) a slot will be idle (BLANKS), contains a single transmission (that is not necessarily successful), or contains a collision. It is clear that when $\varphi_{1,2} = \varphi_{2,1} = \varphi = 0$, the CRA's on the two subnetworks operate independently and therefore the maximal throughput in each subnetwork is 0.429 [9]. Also, when $\varphi_{1,2} = \varphi_{2,1} = \varphi = 1$, the two subnetworks completely overlap, and therefore the two stations always hear the same events and all nodes in both subnetworks follow the same binary-tree. Consequently, the two subnetworks can be viewed as a single station network, and the total maximal throughput will be 0.429. Since the network is symmetric, the maximal throughput per station is 0.2145. We see that even at this extreme point, our approximation is only 6 percent away from the exact value. For values of φ up to 0.5, there is no noticeable difference between our approximation and the simulation results.

Fig. 4 is the same as Fig. 3, except that here the nodes employ the CBTCRA. In this case, when $\varphi_{1,2} = \varphi_{2,1} = \varphi = 0$, the maximal throughput in each station is 0.449. When $\varphi_{1,2} = \varphi_{2,1} = \varphi = 1$ the nodes of the two subnetworks are not following the same binary-tree (because of the possible clipping), therefore the exact value of the maximal throughput in this case is not known (it is not 0.449/2). In both Figs. 3 and 4 the maximal throughput is obtained by choosing an optimal value for $\Delta (= \Delta_1 = \Delta_2)$ for each φ. It is interesting to note that the optimal value of Δ is almost unchanged when φ changes from 0 to 1. This is important since it implies that one can fix Δ even when the value of φ is unknown or is changing with time, and the performance of the network (in terms of maximal throughput) will not change much.

In Fig. 5 we depict the region of attainable throughputs in a two-station network for different values of φ's when the BTCRA is employed. Finally, in Fig. 6, we consider a four-station network employing BTCRA with $\varphi_{1,2} = \varphi_{2,1} = \varphi_{1,3} = \varphi_{3,1} = \varphi_{2,3} = \varphi_{3,2} = 0.2$; $\varphi_{2,4} = \varphi_{4,2} = \varphi_{3,4} = \varphi_{4,3} = 0$; $\varphi_{1,4} = \varphi_{4,1} = \varphi$. We depict the region of attainable throughputs of subnetworks 1 and 4 for different values of φ. In the graph we also indicate the throughput in subnetworks 2 and 3.

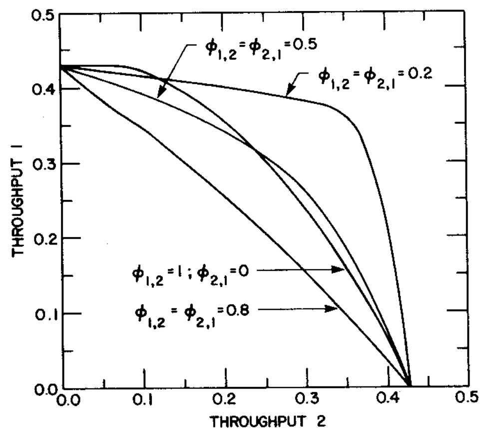


Fig. 5. Regions of attainable throughput (BTCRA).

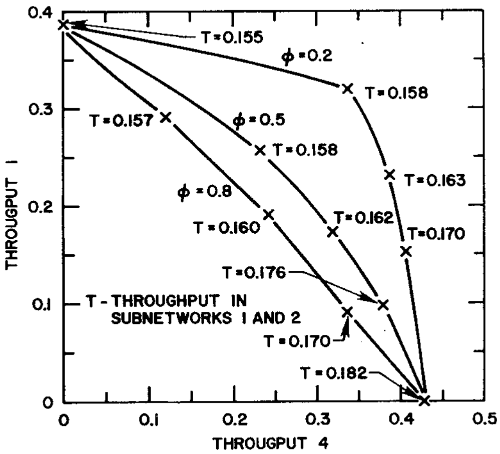


Fig. 6. Regions of attainable throughput (BTCRA).

Appendix

Recursive Computation of $L_i(n, \delta_i, \epsilon_i)$, $L_{j,i}^{(0)}(n, \delta_j, \epsilon_j)$, $L_{j,i}^{(1)}(n, \delta_j, \epsilon_j)$ *and* $M_j(n, \delta_j, \epsilon_j)$

The quantities $L_i(n, \delta_i, \epsilon_i)$, $L_{j,i}^{(0)}(n, \delta_j, \epsilon_j) L_{j,i}^{(1)}(n, \delta_j, \epsilon_j)$, and $M_j(n, \delta_j, \epsilon_j)$ for $n \geq 0$ are computed recursively in a similar manner to that described in [9]. Here we summarize the necessary equations. To simplify, we omit the explicit dependence of all quantities on δ_j and ϵ_j. For instance, $L_{j,i}^{(1)}(n, \delta_j, \epsilon_j)$ is denoted by $L_{j,i}^{(1)}(n)$. As in [9], we have

$$L_j(0) = 1/(1-2\delta_j);\ L_j(1) = [1+\epsilon_j L_j(0)]/(1-\epsilon_j)$$

$$L_{j,i}^{(0)}(0) = 1/(1-2\delta_j);\ L_{j,i}^{(0)}(1) = [1-\varphi_{j,i}+\epsilon_j L_{j,i}^{(0)}(0)]/(1-\epsilon_j);$$

$$L_{j,i}^{(1)}(0) = 0;\ L_{j,i}^{(1)}(1) = \phi_{j,i}/(1-\epsilon_j).$$

In the following

$$Q_j(u, n) = \binom{n}{u} p_j^u (1-p_j)^{n-u} \quad 0 \leq u \leq n$$

$$A_{j,i}^{(0)}(n) = (1-\varphi_{j,i})^n;\ A_{j,i}^{(1)}(n) = n\varphi_{j,i}(1-\varphi_{j,i})^{n-1}$$

Now for $n \geq 2$ we have the following.

BTCRA:

$$L_j(n) = 1 + \sum_{u=0}^{n} Q_j(u, n)[L_j(u) + L_j(n-u)]$$

$$L_{j,i}^{(l)}(n) = A_{j,i}^{(l)} + \sum_{u=0}^{n} Q_j(u, n)[L_{j,i}^{(l)}(u) + L_{j,i}^{(l)}(n-u)] \quad l = 0, 1$$

CBTCRA:

$$L_j(n) = 1 + Q_j(0, n)[L_j(0) + L_j(n)] + Q_j(1, n) \cdot [L_j(1) + L_j(n-1)] + \sum_{u=2}^{n} Q_j(u, n) L_j(u)$$

$$L_{j,i}^{(l)}(n) = A_{j,i}^{(l)}(0) + Q_j(0, n)[L_{j,i}^{(l)}(0) + L_{j,i}^{(l)}(n)] + Q_j(1, n) \cdot [L_{j,i}^{(l)}(1) + L_{j,i}^{(l)}(n-1)] + \sum_{u=2}^{n} Q_j(u, n) L_{j,i}^{(l)}(u) \quad l = 0, 1$$

$$M_j(0) = 0;\ M_j(1) = 1;$$

$$M_j(n) = Q_j(0, n)[M_j(0) + M_j(n)] + Q_j(1, n)[M_j(1) + M_j(n-1)] + \sum_{u=2}^{n} Q_j(u, n) M_j(u) \quad n \geq 2.$$

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