Mobile users: To update or not to update?

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Abstract. Tracking strategies for mobile users in wireless networks are studied. In order to save the cost of using the wireless links mobile users should not update their location whenever they cross boundaries of adjacent cells. This paper focuses on three natural strategies in which the mobile users make the decisions when and where to update: the time-based strategy, the number of movements-based strategy, and the distance-based strategy. We consider both memoryless movement patterns and movements with Markovian memory along a topology of cells arranged as a ring. We analyze the performance of each one of the three strategies under such movements, and show the performance differences between the strategies.

1. Introduction

Future wireless networks will provide ubiquitous communication services to a large number of mobile users. The design of such networks is based on a *cellular* architecture [6–9] that allows efficient use of the limited available spectrum. The cellular architecture consists of a *backbone* network with fixed *base stations* interconnected through a fixed network (usually wired), and of mobile units that communicate with the base stations via wireless links. The geographic area within which mobile units can communicate with a particular base station is referred to as a *cell*. Neighboring cells overlap with each other, thus ensuring continuity of communications when the users move from one cell to another. The mobile units communicate with each other, as well as with other networks, through the base stations and the backbone network.

One of the important issues in cellular networks is the design and analysis of strategies for *tracking* the mobile users. In these networks, whenever there is a need to establish communication with any particular user, the network has first to find out which base station can communicate with that user. This is due to the fact that the users are mobile and could be anywhere within the area covered by the network. This issue was considered in [1,6,2].

In [1] and [6] efficient data structures for manipulating the information regarding the locations of the mobile users were investigated. In these works, it is assumed that this information is sent by the mobile users to the backbone network whenever the users move from cell to cell. The focus in these studies is on the trade-off between updating and retrieving information from directories in the backbone network. Thus, the issue considered in these works is the cost of utilizing the *wired* links of the backbone network for management of directories.

Another important issue is the cost of utilizing the *wireless* links for the actual tracking of mobile users. To illustrate this issue consider the simple tracking strategies known as the Always-Update, in which each mobile user transmits an update message whenever it moves into a new cell, and the Never-Update, in which the users never send update messages regarding their location. Clearly, under the former strategy the overhead due to transmissions of *update* messages is very high, especially in networks with small cells and a large number of highly mobile users, but the overhead for *finding* users is zero since the current location of each user is always known. With the latter strategy there is no overhead for *updating* but whenever there is a need to *find* a particular user, a network-wide search is required, and this overhead is very high.

These two simple strategies demonstrate the basic trade-off that is inherent to the problem. Thus, there are two basic operations that are elemental to tracking mobile users – **update** and **find** (or search/locate). Associated with each one of these operations there is a certain cost. For instance, frequent updates cause high power consumption at the mobile units and load the wireless network. Network-wide searches load both the backbone and the wireless networks. As demonstrated by the two examples above, increasing one cost leads to a decrease in the other one. In fact, the above simple strategies are the two extreme strategies, in which one cost is minimized and the other cost is maximized. Existing systems use either one of these two strategies or the following combination of them (used in current cellular

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telephone networks). Each user is affiliated with some geographic region, referred to as its home-system. Within the home-system, the Never-Update strategy is used. When the user moves into another region, then it must manually register (update), and then the Never-Update strategy is used within that region. The partition into regions is *static*, and is based on the market areas where licenses for cellular systems can be granted to operator companies by the FCC.

Recently, the problem of utilizing efficiently the wireless links was considered in [2], where a new approach according to which a subset of all cells is selected and designated as *reporting cells* is presented. The idea is that mobile users will transmit update messages only upon entering a reporting cell, and a search for any user will always be restricted to the vicinity of a reporting cell – the one to which the user lastly reported. This strategy is *static* in the sense that the location of the reporting centers is fixed. Also, the strategy is *global* in the sense that all mobile users transmit their update messages in the same set of cells. Note that it might happen that a mobile user will transmit frequent update messages, even if it does not move far since it can move in and out a reporting center frequently. It can also happen that a mobile user will not transmit update messages for long periods of time, even if it moves a lot, since it does not enter a reporting center. The question of where to place the reporting centers so that some cost function is optimized is answered in [2] for various cell topologies.

In this paper we focus on *dynamic* strategies in which the mobile users transmit update messages according to their movements and not in predetermined cells. These strategies are *local* in the sense that the location update may differ from user to user and the users themselves decide when and where to transmit their update messages.

The simplest dynamic strategy that is considered is the *time-based* update in which each mobile user updates its location periodically every T units of time, where T is a parameter. Another dynamic strategy considered is the *movement-based* update in which each mobile user counts the number of boundary crossing between cells incurred by its movements and when this number exceeds a parameter M it transmits an update message. The last dynamic strategy considered is the *distance-based* update in which each mobile user tracks the distance it moved (in terms of cells) since last transmitting an update message, and whenever the distance exceeds a parameter D it transmits an update message.

It is not difficult to realize that from an implementation point of view the time-based strategy is the simplest since the mobile users need to follow only their local clocks. The movement-based strategy is more difficult to implement since the mobile users need to be aware when they cross boundaries between cells. The implementation of the distance-based strategy is the most difficult one since the mobile users need information about the topology of the cellular network.

In this paper we consider both memoryless movement patterns and movements with Markovian memory along a topology of cells arranged as a ring. We analyze the performance of each one of the three strategies under such movements, and show the performance differences between the strategies. In the memoryless case, we prove that the distance-based strategy is the best among the three strategies, and that the time-based strategy is the worst one among them. In the Markovian case, we show that this is not always true, as for certain types of movement patterns the time-based strategy is better than the movement-based strategy. In both the memoryless and the Markovian movement patterns, the numerical results obtained show that the difference between the movement-based and the time-based strategies is small, and that this difference vanishes as the update rate decreases. The results also show that the difference between the distance-based strategy and the other two strategies is more significant.

Recently it has come to our attention that a distance-based strategy has been independently considered in [5]. In [5] it is assumed that the users are paged with a high rate. Since whenever the user is paged and found the location of this user is updated, they consider the evolution of the system between two successive pagings. Using dynamic programming approach, an iterative algorithm for computing the optimal value for the parameter D is given.

The paper is organized as follows. In section 2 we define the model. In section 3 we present the analysis and results for the memoryless movement pattern. In section 4 we present the analysis and results for the Markovian movement pattern. Section 5 is devoted for discussion and open problems.

2. The model

Consider a cellular wireless radio system with N cells. Two cells are called *neighboring* cells if a mobile user can move from one of them to another, without crossing any other cell. In this paper we study a ring cellular topology in which cells i and $i+1$ are neighboring cells (in the following all arithmetics with cell indices are modulo N). Thus, a mobile user that is in cell i, can only move to cells $i+1$ or $i-1$ or remain in cell i.

To model the movement of the mobile users in the system we assume that time is slotted, and a user can make at most one move during a slot. The movements will be assumed to be stochastic and independent from one user to another. For each user, we analyze two types of movements – the *i.i.d.* (independent and identically distributed) movement and the *Markovian* movement. In the i.i.d. model, if a user is in cell i at the beginning of a slot, then during the slot it moves to cell $i+1$ with probability p, moves to cell $i-1$ with probability p, or remains in cell i with probability $1-2p$ $(0 < p < 1/2)$, independently of its movements in other slots. In the Markovian model, during each slot a user can be in one of the following three states: (i) The stationary state (S) (ii) The right-move state (R) (iii) The left-move state (L). Assume that a user is in cell i at the beginning of a slot. The movement of the user during that slot depends on the state as follows. If the user is in state S then it remains in cell i, if the user is in state R then it moves to cell $i+1$, and if the user is in state L then it moves to cell $i-1$. Let $X(t)$ be the state during slot t. We assume that $\{X(t), t = 0, 1, 2, \ldots\}$ is a Markov chain with transition probabilities $p_{k,l} = Prob[X(t+1) = l / X(t) = k]$ as follows: $p_{R,R} = p_{L,L} = q$, $p_{L,R} = p_{R,L} = v$, $p_{S,R} = p_{S,L} = p$, $p_{L,S} = p_{R,S} = 1 - q - v$ and $p_{S,S} = 1 - 2p$ (see Fig. 1).

We consider three dynamic update strategies: (i) **Time-based update:** Each mobile user transmits an update message every T slots; (ii) **Movement-based update:** Each mobile user transmits an update message whenever it completes M movements between cells; (iii) **Distance-based update:** Each mobile user transmits an update message whenever the distance (in terms of cells) between its current cell and the cell in which it last reported is D (the act of sending an update message by a user is referred to as *reporting*). We assume that N is at least twice the parameter of the update strategy.

The search strategy for finding a user is simple: Whenever a user is looked for by the system, it is first searched at the cell it last reported to, say i. If it is not found there, then it is searched in cells $i \pm j$ for every $j = 1, 2, \ldots, N/2$, starting with $j = 1$ and continuing upward until it is found. A search in cells $i \pm j$ (for a specific j) is counted as a single operation. We assume that the search operation is fast, and terminates within one slot.

We assume that updates and searches are done at the beginning of slots, such that if a user is both updating and being searched in the same slot then the update operation precedes the search. Movements between cells in any slot are assumed to start after all updates and searches associated with that slot are done, and all movements are completed by the end of the slot. A search is assumed to take place only after the system has reached a steady- state, and the time of search, t_s, is chosen independently of the evolution of the system. This implies that the probability that the system is in a particular state at t_s is the stationary probability of that state.

The performance measures that we consider in this paper are the expected number of update messages per slot transmitted by a user, denoted by $\mathcal{U}_x$, and the expected number of searches necessary to locate a user, denoted by $\mathcal{S}_x$, where $x \in \{D, M, T\}$ denotes the strategy under which the measure is taken, i.e., distance-based, movement-based and time- based, respectively. Obviously, for update and search strategies to be good, both measures should be kept small, as explained in the introduction.

Since the movements of different users are independent, it suffices to calculate the above performance measures for one user. Thus, throughout the analysis presented in the sequel we refer to a particular user (arbitrarily chosen).

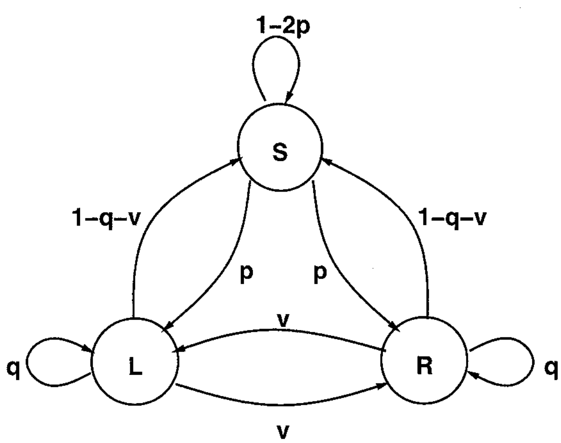

Fig. 1. State diagram of the Markov walk.

3. The I.I.D. model

3.1. Distance-based update

Let $Y(t)$ be the distance between the cell in which the user is located at time t and the cell in which it last transmitted an update message. Notice that here there is no need to distinguish whether the user is on the left or on the right of the cell in which it last transmitted an update message. Clearly, $\{Y(t), t = 0, 1, 2, \ldots\}$ is a Markov chain. Let $Q_d = \lim_{t\to\infty} Prob[Y(t) = d]$, $d = 0, 1, \ldots, D-1$, be the stationary probability distribution of $Y(t)$. The balance equations for these probabilities are

$$2pQ_0 = pQ_1 + pQ_{D-1},$$

$$2pQ_1 = 2pQ_0 + pQ_2,$$

$$2pQ_d = pQ_{d-1} + pQ_{d+1}, \quad 2 \le d \le D-2,$$

$$2pQ_{D-1} = pQ_{D-2}.$$

Solving these equations and normalizing the stationary probabilities to sum to one, we have

$$Q_0 = \frac{1}{D}; \quad Q_i = \frac{2(D-i)}{D^2}, \quad 1 \le i \le D-1,$$

from which it follows that

$$\mathcal{U}_D = pQ_{D-1} = \frac{2p}{D^2}; \quad \mathcal{S}_D = 1 + \sum_{d=1}^{D-1} dQ_d = 1 + \frac{D}{3} - \frac{1}{3D}.$$

The expression for $\mathcal{U}_D$ is due to the fact that an update message is transmitted only when the distance of the user from the cell in which it last reported is $D-1$, and the user moves in the direction that increases this distance (which happens with probability p). The expression for $\mathcal{S}_D$ is true since if at the time of search the distance of the user from the cell in which it last reported is d, then it takes $d+1$ searches until it is found.

3.2. Movement-based update

Since in each slot a user makes a move with probability $2p$ and remains in the same cell with probability $1-2p$, it takes on average $M/2p$ slots to complete M movements. Therefore,

$$\mathcal{U}_M = \frac{2p}{M}\,.$$

The computation of the expected number of searches required to locate a user is more complicated in this case. We start with the fact that given that the user has made m $(0 \le m \le M-1)$ movements since last transmitting an update message at some cell, the probability that its distance from that cell is d is given by

$$Prob[\text{distance} = d|\text{movements} = m] = \begin{cases} 2\binom{m}{\frac{m+d}{2}}\left(\frac{1}{2}\right)^m, & 1 \le d \le m\text{ , m+d even}, \\ 0, & \text{otherwise}. \end{cases} \quad (1)$$

The explanation for (1) is as follows. Assume that the user last reported in cell i. To be at cell $i+d$ after m movements, the number of movements that the user makes towards cell $i+d$ must exceed by exactly d the number of movements that it makes away from that cell. It follows that the number of movements towards cell $i+d$ must be $(m+d)/2$. This accounts for the binomial coefficient. Given that the user moves, the probability that it moves in either one of the two possible directions is $1/2$, which accounts for the term $(1/2)^m$. The factor of 2 is due to the fact that in order to be at distance d the user can be either in $i+d$ or in $i-d$.

Let $L(t) = \max\{\tau \le t|$ the user reported in slot $\tau\}$. Let $I(t)$ be the number of movements that the user has made during the slots $L(t)$, $L(t)+1$, $\ldots$, $t-1$ (if $t-1 < L(t)$ then $I(t) = 0$). Recall that t_s is the slot in which a search occurs. Since a search occurs at random, the probability that it will occur after m movements $(m = 0, 1, 2, \ldots, M-1)$ is uniformly distributed. Therefore, $Prob[I(t_s) = m] = 1/M$ for all $m = 0, 1, 2, \ldots, M-1$ and we have,

$$\mathcal{S}_M = 1 + \frac{1}{M}\sum_{m=0}^{M-1}\sum_{d=1}^{m} d\binom{m}{\frac{m+d}{2}}\left(\frac{1}{2}\right)^{m-1}, \quad (2)$$

where the latter sum is taken only for values of d such that $m+d$ is even. In Appendix A we show that the two sums above simplify to the following:

$$\mathcal{S}_M = 1 + \frac{1}{M}\sum_{m=1}^{M-1} m\left(\frac{1}{2}\right)^{m-1}\binom{m-1}{\lceil\frac{m}{2}\rceil - 1} \quad (3)$$

$$= 1 + \frac{1}{M}\sum_{j=1}^{\frac{M-1}{2}} j\left(\frac{1}{2}\right)^{2(j-1)}\binom{2j}{j} + \left(\frac{1}{2}\right)^M\binom{M}{\frac{M}{2}}\delta_M\,, \quad (4)$$

where $\delta_M = 1$ if M is even and $\delta_M = 0$ otherwise.

3.3. Time-based update

With time-based update we immediately have that

$$\mathcal{U}_T = \frac{1}{T}$$

since the user transmits an update message every T slots. To compute the expected number of searches required to locate the user we use the fact that the probability that the user makes m movements in t slots $(0 \le t \le T-1)$ is given by

$$Prob[\text{movements} = m|\text{slots} = t] = \binom{t}{m}(2p)^m(1-2p)^{t-m}, \quad 0 \le m \le t\,.$$

Using (1) and the fact that $Pr[(t_s mod T) = t] = 1/T$ for all $t = 0, 1, \ldots, T-1$, we obtain

$$\mathcal{S}_T = 1 + \frac{1}{T}\sum_{t=0}^{T-1}\sum_{m=0}^{t}\binom{t}{m}(2p)^m(1-2p)^{t-m} \times \sum_{d=1}^{m} d\binom{m}{\frac{m+d}{2}}\left(\frac{1}{2}\right)^{m-1}, \quad (5)$$

where the latter sum is taken only for values of d such that $m+d$ is even. Changing the order of summation in (5) and using (3) we obtain

$$\mathcal{S}_T = 1 + \frac{1}{T}\sum_{m=0}^{T-1}\sum_{t=m}^{T-1}\binom{t}{m}(2p)^m(1-2p)^{t-m} m \times \left(\frac{1}{2}\right)^{m-1}\binom{m-1}{\lceil\frac{m}{2}\rceil - 1}. \quad (6)$$

This will be used in the next subsection. In Appendix B we show that (5) simplifies to

$$\mathcal{S}_T = 1 + \frac{2}{T}\sum_{j=1}^{T-1}\binom{2j-2}{j-1}\binom{T}{j+1}(-1)^{j+1}p^j\,. \quad (7)$$

3.4. Comparison

In this section we compare the performance of the three strategies analyzed in the previous subsections. We first show that the movement-based strategy is better than the time-based strategy in the sense that for the same expected update rate, the expected number of searches required to locate the user is greater in the time-based strategy. We then show that the distance-based strategy is better than the movement-based strategy in a similar sense.

Since $\mathcal{U}_M = 2p/M$ and $\mathcal{U}_T = 1/T$, it follows that in order to have the same expected update rate, the parameters of the two strategies should satisfy $T = M/2p$.

Proposition 1. For $T = \frac{M}{2p}$, we have $\mathcal{S}_M \le \mathcal{S}_T$.

Proof. Let

$$f(m) = m\left(\frac{1}{2}\right)^{m-1}\binom{m-1}{\lceil \frac{m}{2}\rceil - 1}, \tag{8}$$

$$a(m) = \sum_{t=m}^{T-1}\binom{t}{m}(2p)^{m+1}(1-2p)^{t-m}. \tag{9}$$

From (3) and (6) it follows that

$$\mathcal{S}_M = 1 + \frac{1}{M}\sum_{m=0}^{M-1} f(m),$$

$$\mathcal{S}_T = 1 + \frac{1}{M}\sum_{m=0}^{T-1} a(m)f(m).$$

We first show that $a(m) < 1$. This is true since

$$\begin{aligned} a(m) &= \sum_{i=0}^{T-m-1}\binom{m+i}{m}(2p)^{m+1}(1-2p)^i \\ &< (2p)^{m+1}\sum_{i=0}^{\infty}\binom{m+i}{m}(1-2p)^i \\ &= (2p)^{m+1}\frac{1}{(2p)^{m+1}} = 1, \end{aligned}$$

where the second equality is by [3] (page 199, eq. (5.56)). Thus we have

$$\begin{aligned} \mathcal{S}_M &= 1 + \frac{1}{M}\sum_{m=0}^{M-1} a(m)f(m) + \frac{1}{M}\sum_{m=0}^{M-1}(1-a(m))f(m) \\ &\le 1 + \frac{1}{M}\sum_{m=0}^{M-1} a(m)f(m) + f(M-1)\frac{1}{M}\sum_{m=0}^{M-1}(1-a(m)) \\ &= 1 + \frac{1}{M}\sum_{m=0}^{M-1} a(m)f(m) + f(M-1)\frac{1}{M}\sum_{m=M}^{T-1} a(m) \\ &\le 1 + \frac{1}{M}\sum_{m=0}^{M-1} a(m)f(m) + \frac{1}{M}\sum_{m=M}^{T-1} a(m)f(m) \\ &= \mathcal{S}_T. \end{aligned}$$

The first and the second inequalities follow from the fact that $f(m)$ is a non-decreasing function of m. This can be seen by using (8) to obtain

$$\frac{f(m+1)}{f(m)} = \begin{cases} 1 + 1/m & m \text{ even}, \\ 1 & m \text{ odd}. \end{cases}$$

The second equality follows from the fact that $\sum_{m=0}^{T-1}\frac{a(m)}{M} = 1$. This fact implies that $\frac{1}{M}\sum_{m=M}^{T-1} a(m) = 1 - \frac{1}{M}\sum_{m=0}^{M-1} a(m) = \frac{1}{M}\sum_{m=0}^{M-1}(1-a(m))$. □

We now turn to compare the distance-based strategy with the movement-based strategy. Since $\mathcal{U}_M = 2p/M$ and $\mathcal{U}_D = 2p/D^2$, it follows that in order to have the same expected update rate, the parameters of the two strategies should satisfy $M = D^2$.

Proposition 2. For $M = D^2$,
(a) $\mathcal{S}_D \le \mathcal{S}_M$,
(b) $\lim_{D\to\infty} \mathcal{S}_M/\mathcal{S}_D = \sqrt{8/\pi} \approx 1.56$.

Proof. From (4) we have that

$$\begin{aligned} \mathcal{S}_M &\ge 1 + \frac{1}{D^2}\sum_{j=1}^{\frac{D^2-1}{2}} j\left(\frac{1}{2}\right)^{2(j-1)}\binom{2j}{j} \\ &\ge 1 + \frac{1}{D^2}\sum_{j=1}^{\frac{D^2-1}{2}}\frac{4}{\sqrt{\pi}}e^{-\frac{1}{6}}\sqrt{j} \ge 1 + \frac{1.89}{D^2}\left(1 + \int_1^{\frac{D^2-1}{2}}\sqrt{x}dx\right) \\ &= 1 + \frac{0.63}{D^2}\left(1 + \frac{1}{\sqrt{2}}(D^2-1)^{\frac{3}{2}}\right), \end{aligned} \tag{10}$$

where in the second inequality we used Stirling's bound [3].

Define $g(D) = 1 + (0.63/D^2)(1 + (1/\sqrt{2})(D^2-1)^{3/2})$ and $h(D) = \mathcal{S}_D = 1 + D/3 - 1/3D$. Note that $g(1) = 2.89 > 1 = h(1)$; $g(2) = 3.94 > 3 = h(2)$. Furthermore, $g'(D) \ge h'(D)$ for all $D \ge 2$, which completes the proof of part (a).

Part (b) is obtained from (4) by using Stirling's approximation [3] ($j! \approx \sqrt{2\pi j}(j/e)^j$) and the fact that $\sum_{j=1}^{n}\sqrt{j} \approx \frac{2}{3}n^{3/2}$. □

Part (b) of Proposition 2 indicates that when the update rate decreases, the expected number of searches necessary to locate the user in the movement-based strategy is larger than this number in the distance-based strategy by a factor of about 1.56. One can show equivalently that if the two strategies have the same expected number of searches necessary to locate the user, then when this number increases, the expected update rate in

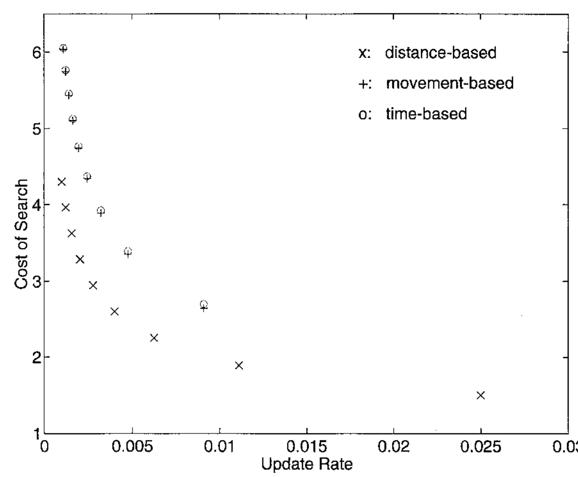

Fig. 2. Cost of search $\mathcal{S}_x$ vs. update rate $\mathcal{U}_x$ in the i.i.d. model for $p = 0.05$.

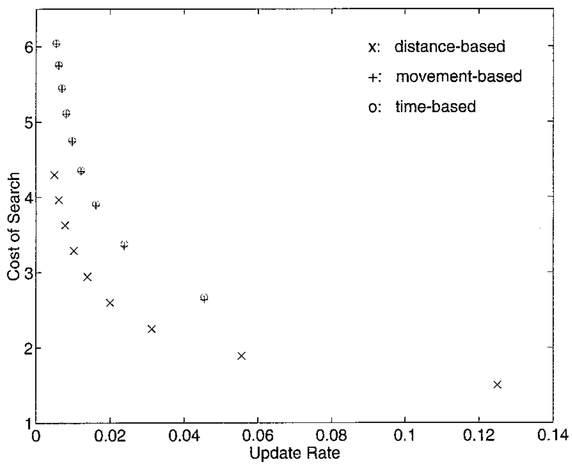

Fig. 3. Cost of search $\mathcal{S}_x$ vs. update rate $\mathcal{U}_x$ in the i.i.d. model for $p = 0.25$.

the movement-based strategy is larger than this number in the distance-based strategy by a factor of about 2.43.

Numerical results

The trade-off between the two costs considered above, i.e., the expected number of update messages per slot transmitted by a user, $\mathcal{U}_x$, and the expected number of searches necessary to locate a user, $\mathcal{S}_x$, is depicted in Figs. 2 and 3. It can be seen that the distance-based strategy is better than the two other strategies. For example, to achieve a cost of search equals to 3, the update rate under the movement-based strategy is more than 2.5 times the update rate under the distance-based strategy (the same is true for the time-based strategy). It is also evident that the difference between the movement-based and the time-based strategies is not significant, and that this difference is vanishing as the update rate decreases.

4. The Markovian model

4.1. Distance-based update

Let $Y(t)$ be the distance between the cell in which the user is located at time t and the cell in which it last transmitted an update message. In the Markovian model it is important to distinguish whether the user is on the right (positive $Y(t)$) or on the left (negative $Y(t)$) of the cell in which it last reported. Clearly, $\{(Y(t), X(t)),\ t = 0, 1, 2, \ldots\}$ is a Markov chain. Let $Q_{d,x} = \lim_{t\to\infty} Prob[Y(t) = d,\ X(t) = x]$, $d = 0, 1, \ldots, D-1$, $x \in \{S, R, L\}$ be the stationary probability distribution of this Markov chain. The balance equations for these probabilities are

$$Q_{0,R} = pQ_{0,S} + qQ_{D-1,R} + vQ_{-(D-1),L} + qQ_{-1,R} + vQ_{1,L},$$

$$2pQ_{0,S} = (1-q-v)(Q_{D-1,R} + Q_{-(D-1),L} + Q_{-1,R} + Q_{1,L}),$$

$$2pQ_{d,S} = (1-q-v)(Q_{d-1,R} + Q_{d+1,L}), \quad 1 \le d \le D-2,$$

$$Q_{d,R} = pQ_{d,S} + qQ_{d-1,R} + vQ_{d+1,L}, \quad 1 \le d \le D-2,$$

$$Q_{d,L} = pQ_{d,S} + vQ_{d-1,R} + qQ_{d+1,L}, \quad 1 \le d \le D-2,$$

$$2pQ_{D-1,S} = (1-q-v)Q_{D-2,R},$$

$$Q_{D-1,R} = pQ_{D-1,S} + qQ_{D-2,R},$$

$$Q_{D-1,L} = pQ_{D-1,S} + vQ_{D-2,R},$$

and from symmetry considerations we have

$$Q_{d,R} = Q_{-d,L}, \quad -(D-1) \le d \le (D-1),$$

$$Q_{d,S} = Q_{-d,S}, \quad 1 \le d \le (D-1).$$

Solving these equations and normalizing the stationary probabilities so that their sum is one we have

$$Q_{d,R} = \frac{p[(D-d)(1-q+v) + 2(q-v)]}{D(1+2p-q-v)[D(1-q+v)+2(q-v)]}, \quad 0 \le d \le D-1,$$

$$Q_{d,L} = \frac{p(D-d)(1-q+v)}{D(1+2p-q-v)[D(1-q+v)+2(q-v)]}, \quad 1 \le d \le D-1,$$

$$Q_{d,S} = \frac{(1-q-v)[(D-d)(1-q+v) + (q-v)]}{D(1+2p-q-v)[D(1-q+v)+2(q-v)]}, \quad 1 \le d \le D-1,$$

$$Q_{0,S} = \frac{1-q-v}{D(1+2p-q-v)}.$$

The user reports at slot t if and only if either $Y(t-1) = D-1$ and $X(t-1) = R$ or $Y(t-1) = -(D-1)$ and $X(t-1) = L$. Therefore $\mathcal{U}_D = 2Q_{D-1,R}$, i.e.,

$$\mathcal{U}_D = \frac{2p(1+q-v)}{D(1+2p-q-v)[D(1-q+v)+2(q-v)]}.$$

Since the number of searches required to locate a user is just

$$\mathcal{S}_D = 1 + \sum_{i=-(D-1)}^{D-1} |i|(Q_{i,S} + Q_{i,R} + Q_{i,L}) = 1 + 2\sum_{i=1}^{D-1} i(Q_{i,S} + Q_{i,R} + Q_{i,L}),$$

we have that

$$\mathcal{S}_D = 1 + \frac{(D-1)[D+1-(q-v)(D-2)]}{3[D-(q-v)(D-2)]}\,.$$

4.2. Movement-based update

Let $Y(t)$ be the distance between the cell in which the user is located in slot t and the cell in which it last transmitted an update message. As before, positive (negative) $Y(t)$ indicates the user is on the right (left) of the cell in which it last transmitted an update message. Clearly, $-(M-1) \le Y(t) \le M-1$. Let $L(t) = \max\{\tau \le t \mid$ the user reported in slot $\tau\}$. Let $I(t)$ be the number of movements that the user has made during the slots $L(t)$, $L(t)+1, \ldots, t-1$ (if $t-1 < L(t)$ then $I(t) = 0$). Clearly, $0 \le I(t) \le M-1$.

To compute the expected number of update messages per slot transmitted by the user, $\mathcal{U}_M$, we focus on the Markov chain $\{(I(t), X(t)), t = 0, 1, 2, \ldots\}$ with the stationary probabilities $Q_{m,x} = \lim_{t\to\infty} Prob[I(t) = m, X(t) = x]$, $m = 0, 1, \ldots, M-1$, $x \in \{S, R, L\}$. The following balance equations will be used shortly.

$$Q_{i-1,R} + Q_{i-1,L} = Q_{i,R} + Q_{i,L}\,, \quad i = 0, 1, \ldots, M-1\,,$$

$$(1-q-v)(Q_{i-1,R} + Q_{i-1,L}) = 2pQ_{i,S}\,, \quad i = 0, 1, \ldots, M-1\,,$$

where $i-1$ is computed modulo M. The user transmits an update message at slot t if and only if $I(t-1) = M-1$ and $X(t-1)$ is either R or L. Therefore,

$$\begin{aligned}\mathcal{U}_M &= Q_{M-1,R} + Q_{M-1,L} = Q_{0,R} + Q_{0,L} = \frac{1}{M} - Q_{0,S} \\ &= \frac{1}{M} - \frac{1-q-v}{2p}(Q_{M-1,R} + Q_{M-1,L}) \\ &= \frac{1}{M} - \frac{1-q-v}{2p}\mathcal{U}_M\,.\end{aligned}$$

The second and fourth equalities follow from the above balance equations. The third equality follows from the fact that $Q_{m,R} + Q_{m,S} + Q_{m,L} = 1/M$ for all m, which follows from symmetry considerations. Solving for $\mathcal{U}_M$ we have

$$\mathcal{U}_M = \frac{2p}{M(1+2p-q-v)}\,.$$

To compute the expected number of searches necessary to locate a user, $\mathcal{S}_M$, we will consider an embedded Markov chain that ignores the states in which the user does not move. To that end, let $J(t, t')$ be the number of movements that the user has made during the slots $L(t), L(t)+1, \ldots, t'-1$. Recall that t_s is the slot in which a search occurs. Let $t_m = \max\{t \ge L(t_s) | J(t_s, t) = m\}$. The embedded Markov chain is $\{(Y(t_m), X(t_m)), m = 0, 1, 2, \ldots\}$.

Let $P_m(d, x|x') = Prob[Y(t_m) = d, X(t_m) = x | X(t_0) = x']$ (note that from the definition of t_m it follows that $Y(t_0) = 0$). Define $\hat{P}_m(d, x|x') = P_m(d, x|x') + P_m(-d, x|x')$ for $d > 0$, and let $\hat{P}_m(d|x') = \hat{P}_m(d, R|x') + \hat{P}_m(d, L|x')$. By symmetry considerations we have that $\hat{P}_m(d|R) = \hat{P}_m(d|L)$, for all $d > 0$ and $m \ge 0$. The probability that a user will be at an absolute distance d (from the cell in which it last transmitted an update message) at time t_m, namely after m movements, is therefore,

$$\hat{P}_m(d) = \hat{P}_m(d|R)Prob[X(t_0) = R] + \hat{P}_m(d|L)Prob[X(t_0) = L] = \hat{P}_m(d|R)\,. \quad (11)$$

Returning now to the original Markov chain, we have

$$\begin{aligned}Prob[Y(t_s) = d] &= \sum_{m=0}^{M-1} Prob[Y(t_s) = d | I(t_s) = m] \cdot Prob[I(t_s) = m] \quad (12) \\ &= \sum_{m=0}^{M-1} \hat{P}_m(d) \cdot \frac{1}{M}\,. \quad (13)\end{aligned}$$

Therefore, the expected number of searches required to locate the user is

$$\mathcal{S}_M = 1 + \frac{1}{M}\sum_{m=0}^{M-1}\sum_{d=1}^{M-1} d\hat{P}_m(d)\,,$$

where the latter sum is taken only for $d + m$ even.

To complete the computation we only need to have the quantities $P_m(d, R|R)$ and $P_m(d, L|R)$ for $0 \le d, m \le M-1$. Defining $\alpha = (1+q-v)/2$ and $\beta = (1-q+v)/2$ we have that,

$$P_0(0, R|R) = \alpha\,,$$

$$P_0(0, L|R) = \beta\,,$$

$$P_0(d, x|R) = 0\,, \quad d \ne 0, x \in \{R, L\}\,,$$

$$P_m(d, R|R) = \alpha P_{m-1}(d-1, R|R) + \beta P_{m-1}(d+1, L|R)\,, \quad m \ge 1\,, \quad -(M-1) \le d \le M-1\,,$$

$$P_m(d, L|R) = \beta P_{m-1}(d-1, R|R) + \alpha P_{m-1}(d+1, L|R)\,, \quad m \ge 1\,, \quad -(M-1) \le d \le M-1\,.$$

It is obvious that these probabilities can be computed recursively from the above relations. Using transform techniques, one can obtain (rather complicated) closed-form expressions for these probabilities.

4.3. Time-based update

As in the i.i.d. case, with time-based update we have

$$\mathcal{U}_T = \frac{1}{T}$$

since the user transmits an update message every T slots. We now compute the expected number of searches required to locate the user. As in the previous subsection, let $L(t) = \max\{\tau \le t \mid \text{the user reported in slot } \tau\}$. Let $Y(t)$ be the distance between the cell in which the user is located in slot t and the cell in which it last reported. Recall that t_s is the slot in which the user is being searched. Then we have

$$\mathcal{S}_T = 1 + \frac{1}{T}\sum_{t=0}^{T-1}\sum_{d=-t}^{t} |d| \; Prob[Y(t_s) = d \mid (t_s \bmod T) = t] .$$

The probabilities $Prob[Y(t_s) = d \mid (t_s \bmod T) = t]$ are determined as follows. Let $Q_t^x(d) = Prob[Y(t_s) = d \mid X(L(t_s)) = x, \; (t_s \bmod T) = t]$. Then for $t = 0, 1, \ldots, T-1$ and $d = -t, -(t-1), \ldots, 0, \ldots, t$, we have,

$$Prob[Y(t_s) = d \mid (t_s \bmod T) = t] = \mu(S) Q_t^S(d) + \mu(R) Q_t^R(d) + \mu(L) Q_t^L(d) ,$$

where $\mu(x)$ is the stationary probability of being in state x, i.e.,

$$\mu(S) = \frac{1-q-v}{1+2p-q-v} ;$$

$$\mu(R) = \mu(L) = \frac{p}{1+2p-q-v} .$$

The probabilities $Q_t^x(d)$ are computed for $t = 0, 1, \ldots, T-1$ and $d = -t, -(t-1), \ldots, 0, \ldots, t$, via the following recursion.

$$Q_t^S(d) = (1-2p)\; Q_{t-1}^S(d) + p\; Q_{t-1}^L(d) + p\; Q_{t-1}^R(d) , \tag{14}$$

$$Q_t^R(d) = (1-q-v)\; Q_{t-1}^S(d-1) + q\; Q_{t-1}^R(d-1) + v\; Q_{t-1}^L(d-1) , \tag{15}$$

$$Q_t^L(d) = (1-q-v)\; Q_{t-1}^S(d+1) + q\; Q_{t-1}^L(d+1) + v\; Q_{t-1}^R(d+1) \tag{16}$$

initiated with

$$Q_0^R(d) = Q_0^L(d) = Q_0^S(d) = \delta(d) ,$$

where $\delta(d) = 1$ if $d = 0$ and $\delta(d) = 0$ otherwise.

The explanation for (14) is as follows. Given that in slot $L(t_s)$ (the slot in which the user last reported) the user is in state S, the user will be in slot $L(t_s) + 1$ in state S with probability $(1-2p)$, in state L with probability p, and in state R with probability p. Given that in slot $L(t_s) + 1$ the user is in state x ($x \in \{S, R, L\}$) and that $(t_s \bmod T) = t$, the probability that in slot t_s the user will be at distance d from the cell in which it last reported is $Q_{t-1}^x(d)$. The explanations for (15) and (16) are similar.

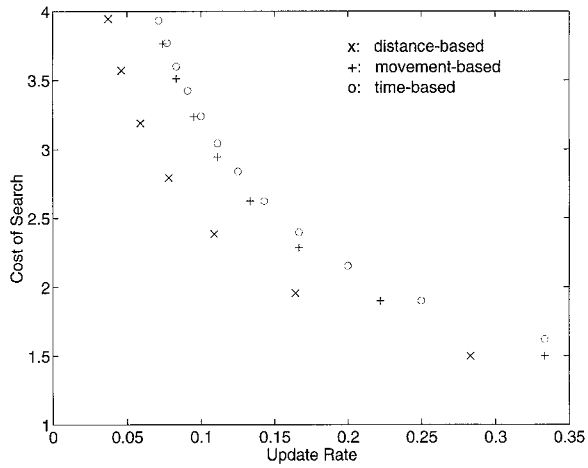

Fig. 4. Cost of search $\mathcal{S}_x$ vs. update rate $\mathcal{U}_x$ in the Markovian model for $q = 0.8, v = 0.1, p = 0.1$.

4.4. Numerical results

The trade-off between the two costs considered above, i.e., the expected number of update messages per slot transmitted by a user, $\mathcal{U}_x$, and the expected number of searches necessary to locate a user, $\mathcal{S}_x$, is depicted in Figs. 4 and 5. As in the i.i.d. case, it can be seen that the distance-based strategy is better than the other two strategies. However, it is interesting to note that unlike in the i.i.d. model, in the Markovian model there are cases in which the time-based strategy is better than the movement-based strategy (see Fig. 5). It seems that this phenomenon occurs when the movement pattern is "restless and erratic", i.e., when p and q are small, and v is close to 1. Note also that as in the i.i.d. case, the difference between the movement-based and the time-based strategies seems to vanish as the update rate decreases.

5. Discussion

In this paper we focus on three natural dynamic tracking strategies, namely, the distance-based, the

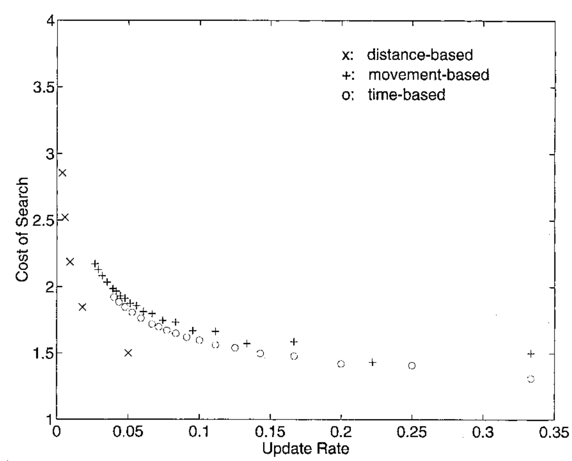

Fig. 5. Cost of search $\mathcal{S}_x$ vs. update rate $\mathcal{U}_x$ in the Markovian model for $q = 0.1, v = 0.8, p = 0.1$.

movement-based and the time-based strategies. We considered a network in which the cells are arranged in a ring topology, and analyzed the performance of each one of the three strategies under a memoryless and a Markovian movement patterns.

In the memoryless case, we proved that the distance-based strategy is the best among the three strategies, and that the time-based strategy is the worst one among them. In the Markovian case, we showed that this is not always true, as for certain types of movement patterns the time-based strategy is better than the movement-based strategy. In both the memoryless and the Markovian movement patterns, the numerical results obtained show that the difference between the movement-based and the time-based strategies is small, and that this difference vanishes as the update rate decreases. The difference between the distance-based strategy and the other two strategies is more significant.

There are a number of problems that are not addressed in this work, which could be the subject of further research. In this paper we considered only a ring topology. However, the ideas and techniques used here seem to be extendable to other topologies, such as a grid graph. Another possible extension could be to consider other walking patterns of the users, and other strategies. In particular, one could adopt a combination of all or some of the strategies considered in this paper. Finally, the cost defined here for the search operation is only one reasonable possibility. It is interesting to see how the results would change (if at all) if this cost is defined differently.

Appendix A

The goal is to simplify (2),

$$\mathcal{S}_M = 1 + \frac{1}{M}\sum_{m=0}^{M-1}\sum_{d=0}^{m} d\binom{m}{\frac{m+d}{2}}\left(\frac{1}{2}\right)^{m-1},$$

where the sum is taken only for values of d such that $m+d$ is even.

Let $m+d=2k$ with k an integer. Then,

$$\begin{aligned}
\mathcal{S}_M &= 1 + \frac{1}{M}\sum_{m=0}^{M-1}\left(\frac{1}{2}\right)^{m-1}\sum_{k=\lceil\frac{m}{2}\rceil}^{m}(2k-m)\binom{m}{k} \\
&= 1 + \frac{1}{M}\sum_{m=0}^{M-1}\left(\frac{1}{2}\right)^{m-1} \\
&\quad\times\left[2m\sum_{k=\lceil\frac{m}{2}\rceil}^{m}\binom{m-1}{k-1} - m\sum_{k=\lceil\frac{m}{2}\rceil}^{m}\binom{m}{k}\right] \\
&= 1 + \frac{1}{M}\sum_{m=0}^{M-1} m\left(\frac{1}{2}\right)^{m-1}\sum_{k=\lceil\frac{m}{2}\rceil}^{m}\left[2\binom{m-1}{k-1} - \binom{m}{k}\right] \\
&= 1 + \frac{1}{M}\sum_{m=0}^{M-1} m\left(\frac{1}{2}\right)^{m-1}\sum_{k=\lceil\frac{m}{2}\rceil}^{m}\left[\binom{m-1}{k-1} - \binom{m-1}{k}\right] \\
&= 1 + \frac{1}{M}\sum_{m=1}^{M-1} m\left(\frac{1}{2}\right)^{m-1}\binom{m-1}{\lceil\frac{m}{2}\rceil - 1}. \qquad (17)
\end{aligned}$$

The last equality follows from the telescopic nature of the second summation. This shows that (2) simplifies to (3). Summing first only even values of m in the above expression, i.e., taking $m=2l$ for an integer l we have

$$\frac{1}{M}\sum_{l=1}^{\frac{M-1}{2}} 2l\left(\frac{1}{2}\right)^{2l-1}\binom{2l-1}{l-1} = \frac{1}{M}\sum_{l=1}^{\frac{M-1}{2}} l\left(\frac{1}{2}\right)^{2l-1}\binom{2l}{l}.$$

Next, summing the odd values of m, i.e., taking $m=2l-1$ for an integer l we have

$$\frac{1}{M}\sum_{l=1}^{\frac{M}{2}}(2l-1)\left(\frac{1}{2}\right)^{2l-2}\binom{2l-2}{l-1} = \frac{1}{M}\sum_{l=1}^{\frac{M}{2}} l\left(\frac{1}{2}\right)^{2l-1}\binom{2l}{l}.$$

The odd case is identical to the even case when M is odd. When M is even, only the odd case contributes to the summation. Thus we obtain that (2) simplifies to (4),

$$\mathcal{S}_M = 1 + \frac{1}{M}\sum_{j=1}^{\frac{M-1}{2}}\left(\frac{1}{2}\right)^{2(j-1)}\binom{2j}{j}j + \left(\frac{1}{2}\right)^{M}\binom{M}{\frac{M}{2}}\delta_M,$$

where $\delta_M = 1$ if M is even and $\delta_M = 0$ otherwise.

Appendix B

The goal is to simplify (5),

$$\begin{aligned}
\mathcal{S}_T = 1 + \frac{1}{T}\sum_{t=0}^{T-1}\sum_{m=0}^{t}\binom{t}{m}(2p)^m(1-2p)^{t-m} \\
\times\sum_{d=0}^{m} d\binom{m}{\frac{m+d}{2}}\left(\frac{1}{2}\right)^{m-1},
\end{aligned}$$

where the sum is taken only for values of d such that $m+d$ is even.

Rearranging (5) we obtain

$$\begin{aligned}
\mathcal{S}_T &= 1 + \frac{2}{T}\sum_{m=1}^{T-1}\sum_{d=1}^{m}\sum_{t=m}^{T-1} d\binom{t}{m}\binom{m}{\frac{m+d}{2}}p^m(1-2p)^{t-m} \\
&= 1 + \frac{2}{T}\sum_{m=1}^{T-1}\sum_{d=1}^{m}\sum_{t=m}^{T-1} d\binom{t}{m}\binom{m}{\frac{m+d}{2}}p^m\sum_{j=0}^{t-m}\binom{t-m}{j}(-2p)^j \\
&= 1 + \frac{2}{T}\sum_{m=1}^{T-1}\sum_{d=1}^{m}\sum_{t=m}^{T-1} d\binom{t}{m}\binom{m}{\frac{m+d}{2}}\sum_{j=m}^{t}\binom{t-m}{j-m}(-2)^{j-m}p^j \\
&= 1 + \frac{2}{T}\sum_{j=1}^{T-1}\sum_{m=1}^{j}\sum_{d=1}^{m}\sum_{t=j}^{T-1} d\binom{m}{\frac{m+d}{2}}\binom{t}{m}\binom{t-m}{j-m}(-2)^{j-m}p^j.
\end{aligned}$$

Since we need the above expression to simplify to (7) we have to show that,

$$\sum_{m=1}^{j}\sum_{d=1}^{m}\sum_{t=j}^{T-1} d\binom{m}{\frac{m+d}{2}}\binom{t}{m}\binom{t-m}{j-m}(-1)^{1-m}2^{j-m}$$
$$=\binom{2j-2}{j-1}\binom{T}{j+1}. \tag{18}$$

To simplify the left hand side of (14) we prove the following identity:

$$\sum_{t=j}^{T-1}\binom{t}{m}\binom{t-m}{j-m}=\binom{j}{m}\binom{T}{j+1}. \tag{19}$$

Proof. Identity (19): By [3] (page 174, eq. (6)) we get that $\binom{t}{m}\binom{t-m}{j-m}=\binom{j}{m}\binom{t}{j}$, and therefore,

$$\sum_{t=j}^{T-1}\binom{t}{m}\binom{t-m}{j-m}=\binom{j}{m}\sum_{t=j}^{T-1}\binom{t}{j}.$$

The claim follows since by [3] (p. 174, eq. (9)) we get that $\sum_{t=j}^{T-1}\binom{t}{j}=\binom{T}{j+1}$. □

Consequently, it remains to show that

$$\sum_{m=1}^{j}\binom{j}{m}(-1)^{1-m}2^{j-m}\sum_{d=1}^{m} d\binom{m}{\frac{m+d}{2}}=\binom{2j-2}{j-1}. \tag{20}$$

We now partition the left hand side of (20) into two cases: In the even case $d=2l$ and $m=2i$ where l and i are integers and in the odd case $d=2l-1$ and $m=2i-1$ where l and i are integers. Thus, the left hand side of (20) becomes

$$-2^j\sum_{i=1}^{\frac{j}{2}}\left(\frac{1}{2}\right)^{2i}\binom{j}{2i}\sum_{l=1}^{i}2l\binom{2i}{i+l}$$
$$+2^j\sum_{i=1}^{\frac{j+1}{2}}\left(\frac{1}{2}\right)^{2i-1}\binom{j}{2i-1}\sum_{l=1}^{i}(2l-1)\binom{2i-1}{i+l-1}.$$

To simplify this expression we need the following two identities. We prove only the first identity, the proof for the second identity follows similar arguments.

$$\sum_{l=1}^{i} l\binom{2i}{i+l}=\frac{i}{2}\binom{2i}{i}, \tag{21}$$

$$\sum_{l=1}^{i}(2l-1)\binom{2i-1}{i+l-1}=\frac{i}{2}\binom{2i}{i}. \tag{22}$$

Proof. Identity (21): First we replace l by $l+i-i$ and get that the left hand side is

$$\sum_{l=1}^{i}(l+i)\binom{2i}{i+l}-i\sum_{l=1}^{i}\binom{2i}{i+l}.$$

Since $(l+i)\binom{2i}{i+l}=2i\binom{2i-1}{i+l-1}$ it follows that the left hand side is

$$2i\sum_{l=1}^{i}\binom{2i-1}{i+l-1}-i\sum_{l=1}^{i}\binom{2i}{i+l}.$$

Since $\sum_{x=1}^{y}\binom{y}{x}=2^y$ and $\binom{y}{x}=\binom{y}{y-x}$, it follows that $\sum_{l=1}^{i}\binom{2i-1}{i+l-1}=\frac{2^{2i-1}}{2}$ and that $\sum_{l=1}^{i}\binom{2i}{i+l}=\frac{1}{2}\left(2^{2i}-\binom{2i}{i}\right)$. Therefore, the left hand side is

$$2i\frac{2^{2i-1}}{2}-i\frac{1}{2}\left(2^{2i}-\binom{2i}{i}\right)=\frac{i}{2}\binom{2i}{i}.$$ □

Using identities (21) and (22), the left hand of (20) becomes

$$2^j\sum_{i=1}^{\frac{j+1}{2}}\left(\frac{1}{2}\right)^{2i} i\binom{j}{2i-1}\binom{2i}{i}-2^j\sum_{i=1}^{\frac{j}{2}}\left(\frac{1}{2}\right)^{2i} i\binom{j}{2i}\binom{2i}{i}.$$

It remains to show that for an even j

$$2^j\sum_{i=1}^{\frac{j}{2}}\left(\frac{1}{2}\right)^{2i} i\binom{2i}{i}\left[\binom{j}{2i-1}-\binom{j}{2i}\right]=\binom{2j-2}{j-1}, \tag{23}$$

and for an odd j

$$2^j\sum_{i=1}^{\frac{j-1}{2}}\left(\frac{1}{2}\right)^{2i} i\binom{2i}{i}\left[\binom{j}{2i-1}-\binom{j}{2i}\right]$$
$$+\frac{j+1}{4}\binom{j+1}{\frac{j+1}{2}}=\binom{2j-2}{j-1}. \tag{24}$$

We only prove (23), the proof for (24) is similar. We need the following two identities. Again since they are similar we are going to prove only the first one.

$$\sum_{i=1}^{\frac{j}{2}}\left(\frac{1}{2}\right)^{2i-1} i\binom{2i}{i}\binom{j}{2i-1}=j\binom{j-\frac{3}{2}}{\frac{j}{2}-1}, \tag{25}$$

$$\sum_{i=1}^{\frac{j}{2}}\left(\frac{1}{2}\right)^{2i-1} i\binom{2i}{i}\binom{j}{2i}=j\binom{j-\frac{3}{2}}{\frac{j}{2}}. \tag{26}$$

Proof. Identity (25): By the definition of $\binom{x}{y}$ it follows that $i\binom{2i}{i}\binom{j}{2i-1}=2j\binom{2i-2}{i-1}\binom{j-1}{2i-2}$. Therefore, the left hand side of (25) is equal to

$$j\sum_{i=1}^{\frac{j}{2}}\left(\frac{1}{2}\right)^{2i-2} i\binom{2i-2}{i-1}\binom{j-1}{2(i-1)}.$$

We now use the following identity [3] (eq. (5.38)),

$$\sum_{k=0}^{\frac{n}{2}}\binom{n}{2k}\binom{2k}{k}2^{-2k}=\binom{n-\frac{1}{2}}{\lfloor\frac{n}{2}\rfloor}.$$

We obtain identity (25) by replacing k with $i-1$ and n by $j-1$. $\square$

Plugging identities (25) and (26) in (23), it remains to prove that

$$j2^{j-1}\left[\binom{j-\frac{3}{2}}{\frac{j}{2}-1}-\binom{j-\frac{3}{2}}{\frac{j}{2}}\right]=\binom{2j-2}{j-1}. \tag{27}$$

Again by the definition of $\binom{x}{y}$ we get $\binom{j-\frac{3}{2}}{\frac{j}{2}-1}-\binom{j-\frac{3}{2}}{\frac{j}{2}}=\frac{1}{j-1}\binom{j-\frac{3}{2}}{\frac{j}{2}}$, and thus it remains to prove that

$$\frac{j2^{j-1}}{j-1}\binom{j-\frac{3}{2}}{\frac{j}{2}}=\binom{2j-2}{j-1}. \tag{28}$$

Proof. Identity (28):

$$\begin{aligned}\binom{2j-2}{j-1}&=\frac{(2j-2)(2j-3)\cdots(j+1)(j)}{(j-1)(j-2)\cdots(2)(1)}\cdot\frac{j-1}{j-1}\\&=\frac{j}{j-1}\cdot\frac{2j-2}{j-1}\cdot\frac{2j-4}{j-2}\cdots\frac{j+2}{\frac{j}{2}+1}\\&\quad\cdot\frac{(2j-3)(2j-5)\cdots(j-1)}{(\frac{j}{2})(\frac{j}{2}-1)\cdots(1)}\\&=\frac{j2^{\frac{j}{2}-1}}{j-1}\cdot\frac{2(j-\frac{3}{2})2(j-\frac{5}{2})\cdots2(\frac{j}{2}-\frac{1}{2})}{(\frac{j}{2})(\frac{j}{2}-1)\cdots(1)}\\&=\frac{j2^{j-1}}{j-1}\cdot\binom{j-\frac{3}{2}}{\frac{j}{2}}.\end{aligned}$$

$\square$

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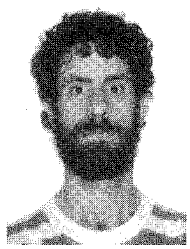

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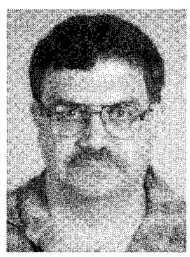

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