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New call blocking versus handoff blocking in cellular networks

Moshe Sidi and David Starobinski
Electrical Engineering Department, Technion, Haifa 32000, Israel

In cellular networks, blocking occurs when a base station has no free channel to allocate to a mobile user. One distinguishes between two kinds of blocking, the first is called new call blocking and refers to blocking of new calls, the second is called handoff blocking and refers to blocking of ongoing calls due to the mobility of the users. In this paper, we first provide explicit analytic expressions for the two kinds of blocking probabilities in two asymptotic regimes, i.e., for very slow mobile users and for very fast mobile users, and show the fundamental differences between these blocking probabilities. Next, an approximation is introduced in order to capture the system behavior for moderate mobility. The approximation is based on the idea of isolating a set of cells and having a simplifying assumption regarding the handoff traffic into this set of cells, while keeping the exact behavior of the traffic between cells in the set. It is shown that a group of 3 cells is enough to capture the difference between the blocking probabilities of handoff call attempts and new call attempts.

1. Introduction

Future wireless networks will provide ubiquitous communication services to a large number of mobile users [4,16,17]. The design of such networks is based on a *cellular* architecture [1,3,7,12,19] that allows efficient use of the limited available spectrum. The cellular architecture consists of a *backbone* network with fixed *base stations* interconnected through a fixed network (usually wired), and of mobile units that communicate with the base stations via wireless links. The geographic area within which mobile units can communicate with a particular base station is referred to a *cell*. Neighboring cells overlap with each other, thus ensuring continuity of communications when the users move from one cell to another. The mobile units communicate with each other, as well as with other networks, through the base stations and the backbone network. A set of channels (frequencies) is allocated to each base station. Neighboring cells have to use different channels in order to avoid intolerable interferences (we do not consider CDMA networks). Many dynamic channel allocation algorithms have been proposed [5,11,20]. These algorithms may improve the performances of the cellular networks. However, for practical reasons, the channel allocation is usually done in a static way. In this work, we will consider only fixed (static) channel assignment.

When a mobile user wants to communicate with another user or a base station, it must first obtain a channel from one of the base stations that hears it (usually, it will be the base station which hears it the best). If a channel is available, it is granted to the user. In the case that all the channels are busy, the new call is blocked. This kind of blocking is called *new call blocking* and it refers to blocking of new calls. The user releases the channel under either of the following scenarios: (i) The user completes the call; (ii) The user moves to another cell before the call is completed. The procedure of moving from one cell to another, while a call is in progress, is called *handoff*. While performing handoff, the mobile unit requires that the base station in the cell that it moves into will allocate it a channel. If no channel is available in the new cell, the handoff call is blocked. This kind of blocking is called *handoff blocking* and it refers to blocking of ongoing calls due to the mobility of the users. An example of new call and handoff call is illustrated in figure 1. The motivation for studying the new call and handoff blocking probabilities is that the Quality of Service (QoS) [2,13] in cellular networks is mainly determined by these two quantities. The first determines the fraction of new calls that are blocked, while the second is closely related to the fraction of admitted calls that terminate prematurely due to dropout. Therefore, a good evaluation of the measures of performance can help a system designer to make its strategic decisions concerning cell size and the number of channel frequencies allocated to each cell.

In this work we present a model that captures the differences between new call blocking and handoff blocking. We consider movements of users along an *arbitrary* topology of cells. Under appropriate statistical assumptions, the system can be modeled as a multi-dimensional continuous-time Markov chain. Multi-dimensional Markov chains usually don't have a product-form solution and are hard to solve even numerically due to the explosion of their state-space. However, we show that in two asymptotic regimes, i.e., for very slow mobile users and for very fast mobile users,

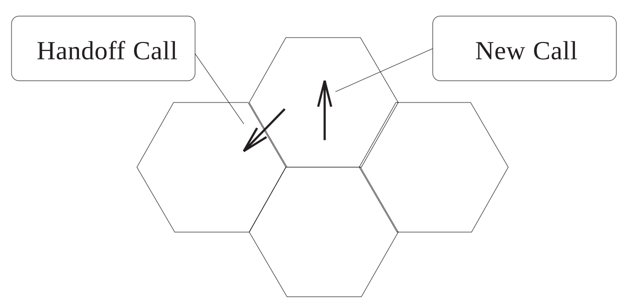

Figure 1. New call and handoff call.

product-form results prevail. For these regimes, we provide expressions for the new call blocking and the handoff blocking probabilities and show the fundamental differences between them for fast mobility.

Next, we introduce an approximation approach that attempts to simplify the solution of the general multi-dimensional Markov chain. The approximation is based on the idea of isolating a set of cells and having a simplifying assumption regarding the handoff traffic into this set of cells. This approach has been used in [6,9,14] where a single cell is isolated and it is assumed that the handoff attempts into this cell are characterized by a Poisson process. The rate of the Poisson process is related to various parameters of the system such as blocking probabilities, mobility of the users, etc. As is shown in [9], when no priority is given to handoff call attempts over new call attempts, no difference exists between these call attempts. In other words, due to the PASTA (Poisson arrivals see time-averages) property, the handoff and the new call blocking probabilities are identical. In the new approximation that we introduce, we isolate a group of cells and make no approximations regarding the handoff traffic between the cells in the group. The handoff traffic into cells of the group from cells outside the group is approximated by a Poisson process. It will be shown that a group of three neighboring cells is enough to differentiate between handoff call attempts and new call attempts. Thus, the underlying Markov chain won't be too complex and results may be easily obtained for any parameters of the system.

The paper is organized as follows. In the next section we describe our model and present the analysis and results for two asymptotic regimes. In section 3, we present our approximation and compare it with prior approximations and with simulations. The last section is devoted to discussion and open problems.

2. The model

2.1. General user motion

Assumptions

We consider a model in which the users move along an arbitrary topology of M cells. Each cell has the same capacity of N channels. In each cell i, new calls are generated according to an independent Poisson process with rate λ_i. Each call holding time T_c, not prematurely dropped, is assumed to be exponential with mean $\overline{T}_c = 1/\mu$. For a new call arrival in a cell, if all N channels in that cell are busy then this arrival is blocked. The fraction of new calls that are blocked or the new call blocking probability in cell i is denoted by P_{B_i}. The sojourn time of every user in a cell T_{h_i} is assumed to be exponential (see [8] for such kind of assumption) with mean $\overline{T}_{h_i} = 1/(\alpha_i\gamma) \equiv 1/\gamma_i$, where α_i is a variable depending on i only. The parameter γ represents the degree of mobility of the users. As users move faster, γ increases. When a call is attempting a handover from cell i then with probability p_{ik} it is to cell k ($\sum_{k\neq i} p_{ik} = 1$). For an on-going call that is attempting handover to another cell, if all N channels in the other cell are busy, then this call is dropped. We denote by $P_{H_{ik}}$ the handoff blocking probability which is the probability of dropout for a call given that this call is attempting handover from cell i to cell k. We denote by P_{T_i} the forced termination probability which is the probability that a call of an admitted user in cell i will terminate due to dropout.

Channel occupancy and blocking probabilities

The above model may be described by an M-dimensional continuous-time Markov chain (CTMC). This is because arrivals of new calls are distributed according to independent Poisson processes, the length of a call is distributed according to a negative exponential distribution and the time that a user stays in a cell is also distributed according to a negative exponential distribution. A simple example of the CTMC for $M = 2$, $N = 3$, $\gamma_1 = \gamma_2 = \gamma$ and $\lambda_1 = \lambda_2 = \lambda$ is shown in figure 2.

To describe the chain we define the vector $\vec{n}$:

$$\vec{n} \triangleq (n_1, n_2, \ldots, n_M).$$

Let $E(\vec{n})$ represent the state where there are n_1 active users in cell 1, n_2 active users in cell 2, …, n_M active users in cell M. For all i, we have $0 \leqslant n_i \leqslant N$ since there are N channels in each cell. The transitions between the states $E(\vec{n})$ correspond to transitions of a continuous-time Markov chain. We denote by $\pi(\vec{n})$ the steady-state probability to find the system in state $E(\vec{n})$. We introduce the following notations:

- $\vec{n}_a^{(i)} \triangleq (n_1, n_2, \ldots, n_i + a, \ldots, n_M)$,
- $\delta^{(i)} \triangleq \begin{cases} 1, & n_i > 0, \\ 0, & n_i = 0, \end{cases}$
- $\beta^{(i)} \triangleq \begin{cases} 1, & n_i < N, \\ 0, & n_i = N. \end{cases}$

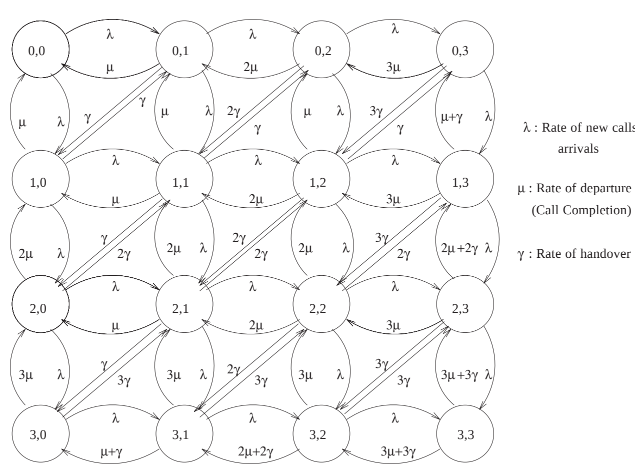

Figure 2. A Markov chain describing 2 cells with 3 channels.

For any $\vec{n}$ the continuous-time Markov chain satisfies the following equilibrium equation:

$$\begin{aligned}
\pi(\vec{n})\sum_{i=1}^{M}\lambda_i\beta^{(i)} + \pi(\vec{n})\sum_{i=1}^{M}n_i(\mu+\gamma_i)
&= \sum_{i=1}^{M}\lambda_i\delta^{(i)}\pi\left(\vec{n}_{-1}^{(i)}\right)\\
&\quad + \sum_{i=1}^{M}\delta^{(i)}\sum_{k=1}^{M}(n_k+1)p_{ki}\gamma_k\beta^{(k)}\pi\left(\vec{n}_{-1,+1}^{(i,k)}\right)\\
&\quad + \sum_{i=1}^{M}\left(1-\beta^{(i)}\right)\sum_{k=1}^{M}(n_k+1)p_{ki}\gamma_k\beta^{(k)}\pi\left(\vec{n}_{+1}^{(k)}\right)\\
&\quad + \sum_{i=1}^{M}(n_i+1)\mu\beta^{(i)}\pi\left(\vec{n}_{+1}^{(i)}\right). \qquad (1)
\end{aligned}$$

The steady-state probabilities $\pi(\vec{n})$ must also satisfy the normalization condition

$$\sum_{n_1,n_2,\ldots,n_M}\pi(\vec{n}) = 1\,. \qquad (2)$$

The left-side of eq. (1) represents the rate of departures from state $E(\vec{n})$. Departures from state $E(\vec{n})$ may occur either when a new call is admitted into the system or when a call leaves a cell (because of handoff or because a call has been completed). The right-side of eq. (1) represents the rate of arrivals into state $E(\vec{n})$. Transitions to state $E(\vec{n})$ may occur from state $E(\vec{n}_{-1}^{(i)})$ ($n_i \neq 0$) when a new call arrives at cell i, or from state $E(\vec{n}_{-1,+1}^{(i,k)})$ ($n_i \neq 0$, $n_k \neq N$) when a successful handoff from cell k to cell i happens, or from state $E(\vec{n}_{+1}^{(k)})$ ($n_i = N$, $n_k \neq N$) when an unsuccessful handoff from cell k to cell i happens, or from state $E(\vec{n}_{+1}^{(i)})$ ($n_i \neq N$) when a call has been completed in cell i.

We obtain the following expressions for P_{B_i} and for $P_{H_{ki}}$:

$$P_{B_i} = \sum_{n_1,n_2,\ldots,n_{i-1},n_{i+1},\ldots,n_M} \times\,\pi(n_1,n_2,\ldots,n_{i-1},N,n_{i+1},\ldots,n_M), \qquad (3)$$

$$P_{H_{ki}} = \left(\sum_{n_1,n_2,\ldots,n_{i-1},n_{i+1},\ldots,n_M} n_k \times\,\pi(n_1,n_2,\ldots,n_{i-1},N,n_{i+1}\ldots,n_M)\right) \times \left(\sum_{n_1,n_2,\ldots,n_M} n_k\pi(n_1,n_2,\ldots,n_M)\right)^{-1}. \qquad (4)$$

Eq. (3) is based on the fact that Poisson arrivals see time averages (PASTA). The probability that a new call that arrives at cell i will be blocked is equal to the sum of all the steady-state probabilities $\pi(\vec{n})$ with $n_i = N$. Eq. (4) for $P_{H_{ki}}$ represents the ratio of the rate of unsuccessful handoffs attempts from cell k to cell i to the total rate of handoff attempts from cell k to cell i (the factor $p_{ki}\gamma_k$, appearing both in the numerator and in the denominator, was cancelled).

Unfortunately, the above Markov chain does not have a product-form solution. Yet, in the next section we will show that exact analytical results can be obtained in two asymptotic regimes – slow mobility and fast mobility.

2.2. Asymptotic regimes

Very slow mobility

When users move very slowly, γ tends to zero. In this case, we obtain from eq. (1) the following equilibrium equation:

$$\begin{aligned}
\pi(\vec{n})\sum_{i=1}^{M}\lambda_i\beta^{(i)} + \pi(\vec{n})\sum_{i=1}^{M}n_i\mu
&= \sum_{i=1}^{M}\lambda_i\delta^{(i)}\pi\left(\vec{n}_{-1}^{(i)}\right)\\
&\quad + \sum_{i=1}^{M}(n_i+1)\mu\beta^{(i)}\pi\left(\vec{n}_{+1}^{(i)}\right). \qquad (5)
\end{aligned}$$

From eq. (5), we see that there is no interaction between the cells. Therefore the distribution of the number of users in each cell corresponds to an $M/M/N/N$ queue. Let $\pi_i(n_i)$ be the marginal probability to find n_i users in cell i. Then

$$\pi_i(n_i) = \frac{\frac{\rho_i^{n_i}}{n_i!}}{\sum_{j=0}^{N}\frac{\rho_i^j}{j!}}\,,$$

where $\rho_i \triangleq \lambda_i/\mu$ is the offered load in cell i.

Now we can derive an expression for the probability $\pi(\vec{n})$ to find the system in state $E(\vec{n})$:

$$\pi(\vec{n}) = \prod_{i=1}^{M}\pi_i(n_i) = \prod_{i=1}^{M}\frac{\frac{\rho_i^{n_i}}{n_i!}}{\sum_{j=0}^{N}\frac{\rho_i^j}{j!}}\,. \qquad (6)$$

Substituting eq. (6) in eqs. (3) and (4) we obtain the following expressions for P_{B_i} and $P_{H_{ki}}$:

$$\begin{aligned}
P_{B_i} &= \sum_{n_1,n_2,\ldots,n_{i-1},n_{i+1},\ldots,n_M}\pi_i(N)\prod_{\substack{j=1\\ j\neq i}}^{N}\pi_j(n_j)\\
&= \pi_i(N) = \frac{\frac{\rho_i^N}{N!}}{\sum_{j=0}^{N}\frac{\rho_i^j}{j!}}\,, \qquad (7)
\end{aligned}$$

$$\begin{aligned}
P_{H_{ki}} &= \frac{\displaystyle\sum_{n_1,n_2,\ldots,n_{i-1},n_{i+1},\ldots,n_M} n_k\pi_i(N)\prod_{\substack{j=1\\ j\neq i}}^{M}\pi_j(n_j)}{\displaystyle\sum_{n_1,n_2,\ldots,n_M} n_k\prod_{j=1}^{M}\pi_j(n_j)}\\
&= \pi_i(N) = \frac{\frac{\rho_i^N}{N!}}{\sum_{j=0}^{N}\frac{\rho_i^j}{j!}}, \qquad (8)
\end{aligned}$$

and we conclude that for very slow mobility environment $P_{H_{ki}} = P_{B_i}$. This result may be explained by the fact that in this environment the cells are statistically quasi-independent.

The ratio $\gamma_i/(\mu+\gamma_i)$ is the probability that a call will need to perform one handoff. In the very slow mobility regime, the probability that a call will need to perform more than one hand-off is negligible. Thus, as long as this probability is very small, i.e., $\gamma_i/(\mu+\gamma_i) \ll 1$, we can approximate the blocking probabilities by the expression given by eq. (7) (or eq. (8) which is identical). Besides that, using eq. (8), we obtain the following approximation for P_{T_i} in the very slow mobility environment:

$$P_{T_i} \approx \frac{\gamma_i}{\mu+\gamma_i}\sum_{k=1}^{M} p_{ik}P_{H_{ik}} \approx \frac{\gamma_i}{\mu}\sum_{k=1}^{M} p_{ik}\frac{\frac{\rho_k^N}{N!}}{\sum_{j=0}^{N}\frac{\rho_k^j}{j!}}. \tag{9}$$

Very fast mobility

When users move very rapidly, γ tends to infinity. Intuitively, we note that in this case there may not be (except for very short periods) more than N users in the network. Let us, first, consider the case of two cells with $N=1$ channel in each cell. Suppose that at a given moment both channels are occupied. Very soon after this moment a handoff will occur, since users move very fast. This handoff will of course be unsuccessful and thus only one active user will stay in the network. In the more general case, suppose that there are $N+1$ users in the network. Then, since the users move very quickly, almost instantaneously one of the users will attempt handover to a cell whose N channels are occupied by the N other users. Since there are only N users in the network, a very large number of handoff attempts succeed. Therefore, a handoff failure is a very rare event and the handoff blocking probability tends to zero (this intuition is formally proved below). However, as we will show, the new call blocking probability does not tend to a zero value in the very fast mobility environment.

We establish now the steady-state probabilities $\pi(\vec{n})$ in the very fast mobility environment. Eq. (1) may be rewritten as follows:

$$\begin{aligned}
&\pi(\vec{n})\sum_{i=1}^{M}\lambda_i\beta^{(i)} + \pi(\vec{n})\sum_{i=1}^{M} n_i\mu \\
&\quad -\sum_{i=1}^{M}\lambda_i\delta^{(i)}\pi\big(\vec{n}_{-1}^{(i)}\big) - \sum_{i=1}^{M}(n_i+1)\mu\beta^{(i)}\pi\big(\vec{n}_{+1}^{(i)}\big) \\
&\quad +\gamma\Bigg[\pi(\vec{n})\sum_{i=1}^{M} n_i\alpha_i \\
&\qquad -\sum_{i=1}^{M}\delta^{(i)}\sum_{k=1}^{M}(n_k+1)p_{ki}\alpha_k\beta^{(k)}\pi\big(\vec{n}_{-1,+1}^{(i,k)}\big) \\
&\qquad -\sum_{i=1}^{M}\big(1-\beta^{(i)}\big)\sum_{k=1}^{M}(n_k+1)p_{ki}\alpha_k\beta^{(k)}\pi\big(\vec{n}_{+1}^{(k)}\big)\Bigg] \\
&\quad = 0.
\end{aligned} \tag{10}$$

In the limit, when $\gamma\to\infty$, the expression between the squared brackets in eq. (10) is equal to 0:

$$\begin{aligned}
&\pi(\vec{n})\sum_{i=1}^{M} n_i\alpha_i \\
&\quad -\sum_{i=1}^{M}\delta^{(i)}\sum_{k=1}^{M}(n_k+1)p_{ki}\alpha_k\beta^{(k)}\pi\big(\vec{n}_{-1,+1}^{(i,k)}\big) \\
&\quad -\sum_{i=1}^{M}\big(1-\beta^{(i)}\big)\sum_{k=1}^{M}(n_k+1)p_{ki}\alpha_k\beta^{(k)}\pi\big(\vec{n}_{+1}^{(k)}\big) \\
&\quad = 0.
\end{aligned} \tag{11}$$

First we consider the cases that $\ell < N$, where $\ell \triangleq \sum_{i=1}^{M} n_i$. For such ℓ's, eq. (11) is simplified to

$$\begin{aligned}
&\pi(\vec{n})\sum_{i=1}^{M} n_i\alpha_i - \sum_{i=1}^{M}\delta^{(i)}\sum_{k=1}^{M}(n_k+1)p_{ki}\alpha_k\pi\big(\vec{n}_{-1,+1}^{(i,k)}\big) \\
&\quad = 0,
\end{aligned} \tag{12}$$

because a transition to state $E(\vec{n})$ due to failure in handoff is not possible.

We claim that the solution of eq. (12), for a vector $\vec{n}$ with $\sum_{j=1}^{M} n_j = \ell$, is given by

$$\pi(\vec{n}) = \pi_\ell\frac{\ell!}{\prod_{j=1}^{M} n_j!}\prod_{j=1}^{M}(P_j)^{n_j}, \tag{13}$$

where the quantities P_i ($1\leqslant i\leqslant M$) are determined via the following equations:

$$P_i\alpha_i = \sum_{k=1}^{M} p_{ki}\alpha_k P_k \quad \text{for } 1\leqslant i\leqslant M, \tag{14}$$

with the normalization condition

$$\sum_{i=1}^{M} P_i = 1. \tag{15}$$

The quantity P_i can be interpreted as the steady-state probability to find a user in cell i. The left-side of eq. (14) can be understood as the rate of departures of a user from cell i and the right-side of eq. (14) as the rate of arrivals of a user to cell i, where the factor γ appearing in both sides of eq. (14) was cancelled. Summing (13) over A_ℓ where

$$A_\ell \triangleq \left\{n_1,n_2,\ldots,n_M \mid \sum_{i=1}^{M} n_i = \ell\right\}$$

and using eq. (15), we obtain that $\sum_{A_\ell}\pi(\vec{n}) = \pi_\ell$. Therefore, we note that π_ℓ is the steady-state probability to find ℓ users in the network. These probabilities will be determined later.

To show the correctness of the claim, i.e., that (13) indeed satisfies eq. (12), we substitute it into the left-side of eq. (12) and using eq. (14), we obtain

$$\pi_\ell \frac{\ell!}{\prod_{j=1}^M n_j!} \prod_{j=1}^M (P_j)^{n_j} \sum_{i=1}^M n_i \alpha_i - \pi_\ell \frac{\ell!}{\prod_{j=1}^M n_j!} \prod_{j=1}^M (P_j)^{n_j} \sum_{i=1}^M \delta^{(i)} \sum_{k=1}^M p_{ki}\alpha_k n_i \frac{P_k}{P_i} = \left(\pi_\ell \frac{\ell!}{\prod_{j=1}^M n_j!} \prod_{j=1}^M (P_j)^{n_j} \right) \times \left(\sum_{i=1}^M n_i\alpha_i - \sum_{i=1}^M n_i\alpha_i \right) = 0.$$

Therefore, (13) satisfies the equilibrium equations when $\ell < N$.

We examine now the cases where $\ell > N$. We will show that π_ℓ, the steady-state probability to find ℓ users in the network, tends to 0 for $\ell > N$. We define E_ℓ as the state where there are ℓ users in the network. In the situation of statistical equilibrium the rate of transitions from E_N to E_{N+1} is equal to the rate of transitions from E_{N+1} to E_N. The rate of transitions from E_N to E_{N+1} is given by

$$\sum_{A_N} \pi(\vec{n}) \sum_{i=1}^M \lambda_i \beta^{(i)}. \tag{16}$$

These transitions are due to new calls which are accepted in the system.

The rate of transitions from E_{N+1} to E_N is

$$\sum_{A_{N+1}} (N+1)\mu\pi(\vec{n}) + \gamma \sum_{A_{N+1}} \pi(\vec{n}) \sum_{i=1}^M \left(1-\beta^{(i)}\right) \sum_{k=1}^M p_{ki}\alpha_k. \tag{17}$$

These transitions are due to calls which leave the system because they have been completed or because they have experienced an unsuccessful handoff.

The equality between expressions (16) and (17), and the fact that γ tends to infinity imply that

$$\pi(\vec{n}) \sum_{i=1}^M \left(1-\beta^{(i)}\right) \sum_{k=1}^M p_{ki}\alpha_k \to 0, \quad \vec{n} \in A_{N+1}. \tag{18}$$

Without loss of the generality, let cell 1 and cell 2 be such that $p_{21} \neq 0$. From (18), we have $\pi(N,1,0,\ldots,0) \to 0$. From eq. (11) it is clear that

$$\pi(\vec{n}) \sum_{i=1}^M n_i\alpha_i \to 0 \Rightarrow \sum_{i=1}^M \delta^{(i)} \sum_{k=1}^M (n_k+1) p_{ki}\alpha_k\beta^{(k)} \pi\left(\vec{n}^{(i,k)}_{-1,+1}\right) \to 0. \tag{19}$$

Substituting $\pi(\vec{n}) = \pi(N,1,0,\ldots,0)$ in (19) we obtain $\pi(N-1,2,0,\ldots,0) \to 0$. Substituting $\pi(\vec{n}) = \pi(N-1,2,0,\ldots0)$ in (19) we see that $\pi(N-2,3,0,\ldots0) \to 0$ and so on. So for all n_1, $N \geqslant n_1 \geqslant 0$, we have $\pi(n_1, N+1-n_1, 0,\ldots,0) \to 0$.

Clearly, there is a cell, say cell 3, such that either $p_{31} \neq 0$ or $p_{32} \neq 0$. If $p_{32} \neq 0$ then beginning with $\pi(n_1, N+1-n_1, 0,\ldots,0) \to 0$ and using $N+1-n_1-n_2$ times the implication of (19), we have $\pi(n_1,n_2,N+1-n_1-n_2,0,\ldots,0) \to 0$. If $p_{31} \neq 0$ then beginning with $\pi(N-n_2+1,n_2,0,0,\ldots,0) \to 0$ and using $N+1-n_2-n_1$ times the implication of (19), we have $\pi(n_1,n_2,N+1-n_1-n_2,0,\ldots,0) \to 0$. Using the same procedure we obtain that for all $\vec{n} \in A_{N+1}$, we have $\pi(\vec{n}) \to 0$. Thus, $\pi_{N+1} \to 0$ since for all $\vec{n} \in A_{N+1}$, $\pi(\vec{n}) \to 0$. Clearly, the rate of transitions from E_{N+1} to E_{N+2} tends to zero, therefore $\pi_{N+2} \to 0$. With the same argument, it can be concluded that for all $\ell \geqslant N+1$ we have $\pi_\ell \to 0$.

We can now determine $\pi(\vec{n})$ for $\vec{n} \in A_N$. We have shown that for all $\vec{n} \in A_N$, $\beta^{(k)}\pi(\vec{n}^{(k)}_{+1}) \to 0$ and thus

$$\sum_{i=1}^M \left(1-\beta^{(i)}\right) \sum_{k=1}^M (n_k+1)p_{ki}\alpha_k\beta^{(k)}\pi\left(\vec{n}^{(k)}_{+1}\right) \to 0.$$

We conclude that for all $\ell \leqslant N$, eq. (11) reduces to eq. (12) and that eq. (12) is satisfied by

$$\pi(\vec{n}) = \pi_\ell \frac{\ell!}{\prod_{j=1}^M n_j!} \prod_{j=1}^M (P_j)^{n_j}.$$

The steady-state probabilities π_ℓ to find the network in state E_ℓ, that is, there are ℓ calls in progress in the system, are determined by considering a simple birth-death process. The equilibrium equation of this birth-death process is

$$\Lambda\pi_\ell = (\ell+1)\mu\pi_{\ell+1}, \quad 0 \leqslant \ell \leqslant N-1, \tag{20}$$

where $\Lambda \triangleq \sum_{i=1}^M \lambda_i$ is the total rate of arrivals in the system. The left-side of eq. (20) represents the rate of transitions from state E_ℓ to state $E_{\ell+1}$. Such transitions are due to new arrivals in the system. The right-side of eq. (20) represents the rate of transitions from state $E_{\ell+1}$ to state E_ℓ. Such transitions occur when users complete their calls and thus leave the system. Solving eq. (20) together with the normalizing condition $\sum_{\ell=0}^N \pi_\ell = 1$, we obtain

$$\pi_\ell = \frac{\frac{\Lambda^\ell}{\mu^\ell \ell!}}{\sum_{i=0}^N \frac{\Lambda^i}{\mu^i i!}}. \tag{21}$$

The final expression for $\pi(\vec{n})$ is then obtained by substituting eq. (21) into eq. (13):

$$\pi(\vec{n}) = \frac{\frac{\Lambda^\ell}{\mu^\ell}}{\sum_{j=0}^N \frac{\Lambda^j}{\mu^j j!} \prod_{j=1}^M n_j!} \prod_{j=1}^M (P_j)^{n_j} \quad \text{for } 0 \leqslant \ell \leqslant N. \tag{22}$$

The derivation of the new call blocking probability at cell i, P_{B_i}, is now straightforward:

$$P_{B_i} = \sum_{n_1,n_2,\ldots,n_{i-1},n_{i+1},\ldots,n_M} \times \pi(n_1,n_2,\ldots,n_{i-1},N,n_{i+1},\ldots,n_m)$$

$$= \pi(0,0,\dots,0,N,0,\dots,0) + 0$$
$$= \frac{\frac{\Lambda^N}{\mu^N N!}}{\sum_{j=0}^{N} \frac{\Lambda^j}{\mu^j j!}} (P_i)^N . \tag{23}$$

We observe that the blocking probability P_{B_i} is *strictly* positive. The handoff blocking probability,

$$P_{H_{ki}} = \left(\sum_{n_1,n_2,\dots,n_{i-1},n_{i+1},\dots,n_M} n_k \times \pi(n_1,n_2,\dots,n_{i-1},N,n_{i+1}\dots,n_M) \right) \times \left(\sum_{n_1,n_2,\dots,n_M} n_k \pi(n_1,n_2,\dots,n_M) \right)^{-1},$$

tends, as expected, to 0 because for all $n_k \geqslant 1$,

$$\pi(n_1,\dots,n_{i-1},N,n_{i+1}\dots,n_M) \to 0.$$

This shows a fundamental difference between the new call blocking probability and the handoff blocking probability in the very fast mobility regime.

We learnt recently that a similar result has been presented in [18].

2.3. Homogeneous traffic systems

Motivation

The goal of this section is to present special results for homogeneous traffic systems, including the forced termination probability P_T. Note that the computation of P_T for non-homogeneous systems is still an open problem. First, we give the mathematical definitions to homogeneous traffic systems. Then, using the results of section 2.2, we compute the blocking probabilities in the two asymptotic regimes. The computation of P_T in the very fast mobility regime can be carried due to the fact that in homogeneous traffic systems we have $P_i = 1/M$ for all i, as is shown in the sequel.

Definitions

We will say that the traffic is homogeneous when: (i) the rate of new call arrivals is identical in each cell, and (ii) the rate of handoff arrivals and departures are equal and identical in each cell. Mathematically, conditions (i) and (ii) can be formulated as follows:

$$\lambda_i = \lambda \quad \forall i, \tag{24}$$
$$\gamma_i = \gamma \quad \forall i, \tag{25}$$
$$\sum_{k=1}^{M} p_{ki} = 1 \quad \forall i. \tag{26}$$

Since the homogeneous traffic system is only a particular case of the general model, it is described by the same M-dimensional continuous-time Markov chain that has been introduced in section 2.1.

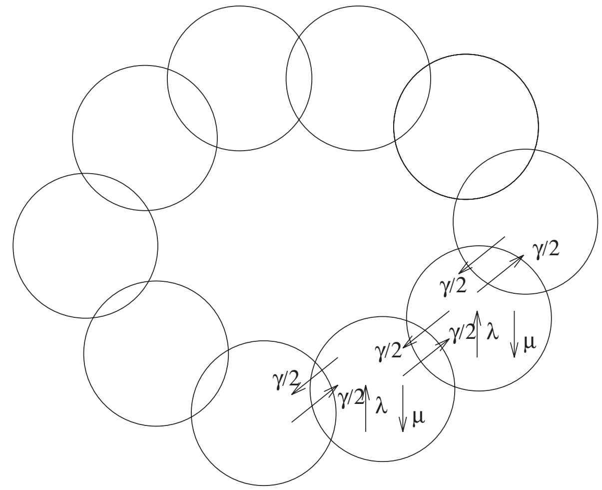

Figure 3. A ring with 10 cells.

An example of a system with homogeneous traffic consists of a ring of M cells (see figure 3).

New calls are generated according to a Poisson process with rate λ in each cell. The sojourn time of every user in a cell T_h is exponential with mean $\overline{T}_h = 1/\gamma$. A call may attempt handover to its left neighbor with probability p and to its right neighbor with probability $1-p$ (in figure 3, $p = 0.5$). Each call holding time T_c, not prematurely dropped, is assumed to be exponential with mean $\overline{T}_c = 1/\mu$. It is trivial to see that this model satisfies conditions (24), (25) and (26).

Blocking probabilities

(i) *Very slow mobility regime.* In the very slow mobility regimes, we obtain from eq. (6)

$$\pi(\vec{n}) = \prod_{i=1}^{M} \pi_i(n_i) = \prod_{i=1}^{M} \frac{\frac{\rho^{n_i}}{n_i!}}{\sum_{j=0}^{N} \frac{\rho^j}{j!}}, \tag{27}$$

where $\rho \triangleq \lambda/\mu$ is the offered load which is identical for each cell. The expressions for P_{B_i} and for $P_{H_{ki}}$ are derived in the same manner as in section 2.1:

$$P_{B_i} = \sum_{n_1,n_2,\dots,n_{i-1},n_{i+1},\dots,n_M} \pi_i(N) \prod_{\substack{j=1\\ j\neq i}}^{N} \pi_j(n_j) = \pi_i(N) = \frac{\frac{\rho^N}{N!}}{\sum_{j=0}^{N} \frac{\rho^j}{j!}}, \tag{28}$$

$$P_{H_{ki}} = \frac{\sum_{n_1,n_2,\dots,n_{i-1},n_{i+1},\dots,n_M} n_k \pi_i(N) \prod_{\substack{j=1\\ j\neq i}}^{M} \pi_j(n_j)}{\sum_{n_1,n_2,\dots,n_M} n_k \prod_{j=1}^{M} \pi_j(n_j)} = \pi_i(N) = \frac{\frac{\rho^N}{N!}}{\sum_{j=0}^{N} \frac{\rho^j}{j!}}, \tag{29}$$

and we conclude that for very slow mobility environment $P_{H_{ki}} = P_{B_i}$, as in the general system. Furthermore, the blocking probabilities do not depend on i and k.

(ii) *Very fast mobility regime.* In the homogeneous traffic system, the steady-state probability P_i that a user is in cell i can be easily found. First, we recall eqs. (14) and (15):

$$P_i = \sum_{k=1}^{M} p_{ki} P_k, \quad \text{for } 1 \leqslant i \leqslant M; \tag{30}$$

$$\sum_{i=1}^{M} P_i = 1. \tag{31}$$

We define the transition matrix $\boldsymbol{B}$ as consisting of elements p_{ik}, that is,

$$\boldsymbol{B} = [p_{ik}]. \tag{32}$$

Moreover, if we define the vector $\boldsymbol{p} = [P_1, P_2, \ldots, P_M]$, eq. (30) may be rewritten:

$$\boldsymbol{p} = \boldsymbol{B}\boldsymbol{p}. \tag{33}$$

The matrix $\boldsymbol{B}$ is a double-stochastic matrix due to condition (26) and due to the fact that $\sum_{k=1}^{M} p_{ik} = 1$ is always true. The unique solution to eqs. (33) and (31) is therefore

$$P_i = \frac{1}{M} \quad \text{for } 1 \leqslant i \leqslant M. \tag{34}$$

Now, we can obtain, from eqs. (22) and (34), the steady-state probabilities $\pi(\vec{n})$ in the very fast mobility regime:

$$\pi(\vec{n}) = \frac{\frac{\Lambda^\ell}{\mu^\ell}}{\sum_{i=0}^{N} \frac{\Lambda^i}{\mu^i i!} \prod_{j=1}^{M} n_j!} \left(\frac{1}{M}\right)^{\ell} \quad \text{for } 0 \leqslant \ell \leqslant N, \tag{35}$$

where

$$\Lambda \triangleq \sum_{i=1}^{M} \lambda_i = M\lambda \quad \text{and} \quad \ell \triangleq \sum_{i=1}^{M} n_i.$$

For $\ell > N$, $\pi(\vec{n}) = 0$. The new call blocking probability $P_{B_i} \equiv P_B$ is identical for each cell:

$$P_B = \frac{\frac{\Lambda^N}{\mu^N N!}}{\sum_{i=0}^{N} \frac{\Lambda^i}{\mu^i i!}} \left(\frac{1}{M}\right)^{N}. \tag{36}$$

The handoff blocking probability tends to zero, as in the general traffic system.

Forced termination probability

(i) *Very slow mobility regime.* We recall that the general expression for P_{T_i}, in the very slow mobility regime, is

$$P_{T_i} \approx \frac{\gamma_i}{\mu} \sum_{k=1}^{M} p_{ik} P_{B_k}. \tag{37}$$

By substituting conditions (25) and (26) and eq. (28) for P_{B_k} (all valid for homogeneous traffic systems) in eq. (37), we derive the following approximation for P_{T_i}:

$$P_{T_i} \approx \frac{\gamma}{\mu} \frac{\frac{\rho^N}{N!}}{\sum_{j=0}^{N} \frac{\rho^j}{j!}}. \tag{38}$$

We conclude that for very slow mobility, P_{T_i} does not depend on i.

(ii) *Very fast mobility regime.* Let π'_k be the probability that a new admitted call finds the system with k other active users. Let $P_{D_{k+1}}$ be the probability that a call will prematurely finish given that there are $k+1$ active users (including the new one) in the system. This probability is the same at any point of time due to the memoryless property of the distribution of the calls length. We note, also, that $P_{D_{k+1}}$ is identical for a new call and for older calls since all the calls are identical (each user can be in each cell with the same probability). Thus, in the very fast mobility regime $P_{T_i} \equiv P_T$ does not depend on i. P_T may be expressed in the following way:

$$P_T = \sum_{k=0}^{N} \pi'_k P_{D_{K+1}}. \tag{39}$$

The probability π'_k that an admitted user will find k other active users in the system is found as follows. First, we denote by A the event that a user is admitted. We have

$$P(A|k) = 1 \quad \text{for } k = 0, 1, 2, \ldots, N-1, \tag{40}$$

$$P(A|N) = 1 - \left(\frac{1}{M}\right)^{N}, \tag{41}$$

where eq. (41) may be understood in the way that when there are N users in the system, a new call may be blocked in a specific cell only if at the same time all the N users are in the same cell. Second, using Bayes theorem, we obtain

$$\pi'_k = P(k|A) = \frac{P(A|k)\pi_k}{P(A)}. \tag{42}$$

Last, substituting $P(A) = (1 - P_B)$ in eq. (42), we obtain

$$\pi'_k = \frac{\pi_k}{1 - P_B} \quad \text{for } k = 0, 1, 2, \ldots, N-1, \tag{43}$$

$$\pi'_N = \frac{\pi_N - P_B}{1 - P_B}. \tag{44}$$

P_{D_K} is found with the following recursive equations:

$$P_{D_1} = \left(\frac{\Lambda}{\Lambda + \mu}\right) P_{D_2}, \tag{45}$$

$$P_{D_k} = \left(\frac{\Lambda}{\Lambda + k\mu}\right) P_{D_{k+1}} + \left(\frac{(k-1)\mu}{\Lambda + k\mu}\right) P_{D_{k-1}} \quad \text{for } N > k > 1, \tag{46}$$

$$P_{D_N} = \left(\frac{\Lambda'}{\Lambda' + N\mu}\right) P_{D_{N+1}} + \left(\frac{(N-1)\mu}{\Lambda' + N\mu}\right) P_{D_{N-1}}, \tag{47}$$

$$P_{D_{N+1}} = \frac{1}{N+1} + \frac{N}{N+1} P_{D_N}, \tag{48}$$

where $\Lambda \triangleq M\lambda$ and

$$\Lambda' \triangleq M\lambda\left(1 - \left(\frac{1}{M}\right)^N\right).$$

The left-side of eq. (46), P_{D_k}, represents the probability that a user (say user X) will be forced to terminate its call before its natural completion, given that there are a total of k users in the system. This probability is equal to the sum of the following probabilities (right-side of eq. (46)): (i) the probability that a user establishes a new call before any of the k active users being in the system has completed its call and then with probability $P_{D_{k+1}}$ user X will terminate prematurely, and (ii) the probability that one of the $k-1$ other active users will complete its call before user X and before any user establishes a new call, and then with probability $P_{D_{k-1}}$ user X will terminate prematurely (when $K = 1$, this probability is of course zero, thus eq. (45)), (iii) the probability that user X will complete its call before any of the $k-1$ other active users in the system and before a new call is established. Then, since user X has successfully completed its call, the probability of forced termination for it is, of course, zero.

Concerning eq. (47) we note that when there are N users in the system, the rate of arrivals of new users in the system is only

$$\Lambda\left(1 - \left(\frac{1}{M}\right)^N\right)$$

since, as explained previously, a fraction $(1/M)^N$ of new calls is blocked. We know from the analysis of the previous section that when there are $N+1$ users in the system, one of them will fail instantaneously in a handoff attempt. Since each user can initially be in each cell with the same probability, each one has the same probability to fail. Eq. (48) states that with probability $1/(N+1)$ user X will encounter a forced termination and with probability $N/(N+1)$ it will be forced to terminate with probability P_{D_N}.

Claim 1. For $k \leqslant N$, P_{D_k} is related to P_{D_1} by the following relation:

$$P_{D_k} = \frac{P_{D_1}}{\Lambda^{k-1}} \sum_{i=0}^{k-1} \frac{(k-1)!}{(k-1-i)!} \Lambda^{k-i-1} \mu^i. \tag{49}$$

Proof. By induction (see appendix A). □

From eqs. (47)–(49) we have

$$P_{D_{N+1}} = \frac{(\Lambda' + N\mu)P_{D_N} - (N-1)\mu P_{D_{N-1}}}{\Lambda'},$$

$$P_{D_{N+1}} = \frac{1}{N+1} + \frac{N}{N+1} P_{D_N},$$

$$P_{D_N} = \frac{P_{D_1}}{\Lambda^{N-1}} \sum_{i=0}^{N-1} \frac{(N-1)!}{(N-1-i)!} \Lambda^{N-1-i} \mu^i,$$

$$P_{D_{N-1}} = \frac{P_{D_1}}{\Lambda^{N-2}} \sum_{i=0}^{N-2} \frac{(N-2)!}{(N-2-i)!} \Lambda^{N-2-i} \mu^i,$$

and we obtain the following expression for P_{D_1}:

$$P_{D_1} = \Lambda'\left([\Lambda' + (N+1)N\mu] \sum_{i=0}^{N-1} \mathrm{A}_i^{N-1} \xi^i - (N+1)(N-1)\mu \sum_{i=0}^{N-2} \mathrm{A}_i^{N-2} \xi^i\right)^{-1}, \tag{50}$$

where $\mathrm{A}_p^n = n!/(n-p)!$ and $\xi = \mu/\Lambda$.

Thus, given the parameters λ, μ, N and M, the value of P_T is calculated using eqs. (50), (49), (48), (44), (43) and (39).

Comparison with a special case

In [10] a special case where N=1 and $M = 2$ was analyzed (2 cells with one channel each). In this case analytical expressions are provided, for any γ, for P_B, P_H and P_T:

$$P_B = \left(\rho + \frac{\rho^2}{1+\Gamma}\right) \frac{1}{1 + 2\rho + \frac{\rho^2}{1+\Gamma}}, \tag{51}$$

$$P_H = \frac{\frac{\rho^2}{1+\Gamma}}{\rho + \frac{\rho^2}{1+\Gamma}}, \tag{52}$$

$$P_T = \frac{1}{1 + \frac{1}{\Gamma P_H}}, \tag{53}$$

where $\rho \triangleq \lambda/\mu$ and $\Gamma \triangleq \gamma/\mu$. When γ tends to zero, we obtain

$$\lim_{\gamma \to 0} P_B = \frac{\rho}{1+\rho}, \tag{54}$$

$$\lim_{\gamma \to 0} P_H = \frac{\rho}{1+\rho}. \tag{55}$$

It is easy to see that eqs. (54) and (55) are respectively identical to eqs. (28) and (29). When γ tends to infinity, we obtain

$$\lim_{\gamma \to \infty} P_B = \frac{\rho}{1+2\rho}, \tag{56}$$

$$\lim_{\gamma \to \infty} P_H = 0, \tag{57}$$

$$\lim_{\gamma \to \infty} P_T = \frac{\rho}{1+\rho}. \tag{58}$$

Eq. (56) is identical to eq. (36) when $N = 1$. Eq. (57) tends to zero, as expected, for large values of γ. From eq. (50) we have $P_{T_1} = \rho/(2+\rho)$. Using eqs. (49), (48), (44), (43) and (39), we obtain $P_T = \rho/(1+\rho)$ which is equal to eq. (58).

3. An approximation

3.1. Motivation and model

Since the state-space of the problem under consideration is very large even for moderate values for the number of cells M and the number of channels N, one has to resort to approximations in order to obtain results for intermediate mobility regimes. The approach presented in [9] to approximate the blocking probabilities is based on isolating a single cell and approximating the handoff traffic into this cell. Our analysis of the asymptotic regime of very fast users showed that P_B and P_H behave differently. The goal of our approximation is to capture as much as we can of this difference. To that end, instead of trying to isolate a single cell as in [9], we suggest to isolate a group of neighboring cells (see figure 4).

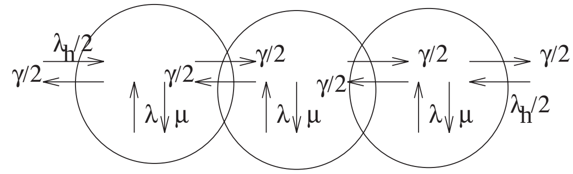

Figure 4. An "isolated" group of 3 cells.

For simplicity, we consider movements of users along a topology of cells arranged as a ring (beltway). The new call arrivals follow an independent Poisson process with rate λ in each cell, the call holding time is distributed exponentially with mean $1/\mu$. The sojourn time of every user in a cell is assumed to be exponential with mean $1/\gamma$. A call may attempt handover to its left neighbor with the same probability as to its right neighbor. Due to the symmetry of the network we have for each cells i and k, $P_{T_i} \equiv P_T$, $P_{B_i} \equiv P_B$ and $P_{H_{ik}} \equiv P_H$. The approximation assumption regarding the handoff traffic is imposed only at the boundaries of the isolated group. Thus, for a ring network, the boundaries consist of two cells, the rightmost cell and the leftmost cell of the group. We assume that the handoff traffic into each of these cells from cells that are not in the group is characterized by an independent Poisson process with mean $\lambda_h/2$ where λ_h is determined in the following way. The average rate at which new calls are carried in each cell is $\lambda(1-P_B)$. The probability that an accepted call will attempt one handoff is $P_r = \gamma/(\mu+\gamma)$. The probability that an accepted call will attempt a second handoff is $P_r^2(1-P_H)$. The probability that an accepted call will attempt a kth handoff is $P_r^k(1-P_H)^{k-1}$. Thus,

$$\lambda_h = \lambda(1-P_B)\sum_{k=1}^{\infty} P_r^k(1-P_H)^{k-1} = \frac{P_r(1-P_B)}{1-P_r(1-P_H)}\lambda. \tag{59}$$

We give, next, the set of nolinear equations which, together with eq. (59), allow us to give an approximation for P_H and P_B.

The group of neighboring cells that we consider consists of K cells. Cell 1 and cell K are respectively the leftmost cell and the rightmost cell of the group. The external handoff traffic flows to these two cells. We define the vector $\vec{n}$:

$$\vec{n} \triangleq (n_1, n_2, \ldots, n_K).$$

We define the set R: $R \triangleq \{1, K\}$. Let $E(\vec{n})$ represent the state where there are n_1 active users in cell 1, n_2 active users in cell 2, …, n_K active users in cell K. For all i, we have $0 \leqslant n_i \leqslant N$ since there are N channels in each cell. The transitions between the states $E(\vec{n})$ correspond to transitions of a continuous-time Markov chain. This is because arrivals of new calls in each cell and arrival of handoff traffic in the cells situated at the boundary of the group are distributed according to independent Poisson processes, the length of a call is distributed according to a negative exponential distribution and the time that a user stays in a cell is also distributed according to a negative exponential distribution. We denote by $\pi(\vec{n})$ the steady-state probability to find the system in state $E(\vec{n})$. For any $\vec{n}$, the continuous-time Markov chain satisfies the following equilibrium equation:

$$\begin{aligned}
&\sum_{i=1}^{K}\lambda\beta^{(i)}\pi(\vec{n}) + \sum_{i\in R}\frac{\lambda_h}{2}\beta^{(i)}\pi(\vec{n}) + \sum_{i=1}^{K} n_i(\mu+\gamma)\pi(\vec{n}) \\
&\quad = \sum_{i=1}^{K}\lambda\delta^{(i)}\pi\big(\vec{n}_{-1}^{(i)}\big) + \sum_{i\in R}\frac{\lambda_h}{2}\delta^{(i)}\pi\big(\vec{n}_{-1}^{(i)}\big) \\
&\qquad + \sum_{i=1}^{K}\delta^{(i)}\sum_{k=1}^{K}(n_k+1)p_{ki}\gamma\beta^{(k)}\pi\big(\vec{n}_{-1,+1}^{(i,k)}\big) \\
&\qquad + \sum_{i=1}^{K}\big(1-\beta^{(i)}\big)\sum_{k=1}^{K}(n_k+1)p_{ki}\gamma\beta^{(k)}\pi\big(\vec{n}_{+1}^{(k)}\big) \\
&\qquad + \sum_{i=1}^{K}(n_i+1)\mu\beta^{(i)}\pi\big(\vec{n}_{+1}^{(i)}\big) \\
&\qquad + \sum_{i\in R}(n_i+1)\frac{\gamma}{2}(1-\beta^{(i)})\pi\big(\vec{n}_{+1}^{(i)}\big)\,,
\end{aligned} \tag{60}$$

where

$$p_{ki} = \begin{cases} 1/2, & \text{for } |i-k| = 1, \\ 0, & \text{otherwise.} \end{cases}$$

The steady-state probabilities $\pi(\vec{n})$ must also satisfy the normalization condition

$$\sum_{n_1,n_2,\ldots,n_K}\pi(\vec{n}) = 1\,. \tag{61}$$

Eq. (60) is very similar to eq. (1). The left-side of (60) represents the rate of departures from the state $E(\vec{n})$. Departures may occur either when a new call is admitted into the system, or when a call leaves a cell (because of handoff or because a call has been completed), or when an handoff call is admitted in one of the two extreme cells. The right-side of (60) represents the rate of arrivals into state $E(\vec{n})$. Transitions to state $E(\vec{n})$ may occur from state $E(\vec{n}^{(i)}_{-1})$ $(n_i \neq 0)$ when a new call arrives at cell i, or from state

$$E\left(\vec{n}^{(i)}_{-1}\right) \quad (i \in R,\ n_i \neq 0)$$

when a handoff call arrives, or from state

$$E\left(\vec{n}^{(i-1,i)}_{-1,+1}\right) \quad (i \geqslant 2,\ n_{i-1} \neq 0,\ n_i \neq N)$$

(resp. $E(\vec{n}^{(i-1,i)}_{+1,-1})$ $(i \geqslant 2,\ n_{i-1} \neq N,\ n_i \neq 0)$) when a successful handoff from cell i (resp. $i-1$) to cell $i-1$ (resp. i) happens, or from state

$$E\left(\vec{n}^{(i)}_{+1}\right) \quad (i \geqslant 2,\ n_{i-1} = N,\ n_i \neq N)$$

(resp. $E(\vec{n}^{(i-1)}_{+1})$ $(i \geqslant 2,\ n_{i-1} \neq N,\ n_i = N)$) when an unsuccessful handoff from cell i (resp. $i-1$) to cell $i-1$ (resp. i) happens, or from state $E(\vec{n}^{(i)}_{+1})$ $(n_i \neq N)$ when a call has been completed in cell i, or from state $E(\vec{n}^{(i)}_{+1})$ $(i \in R,\ n_i \neq N)$ when a call makes a handoff from an extreme cell of the group to the exterior of the group.

The new call blocking probability experimented by users in the cell located at the *middle* of the group, i.e., cell $\lceil \frac{K}{2} \rceil$, serves to approximate P_B. Thus,

$$P_B = \sum_{n_1,n_2,\ldots,n_{\lceil \frac{K}{2}\rceil-1},n_{\lceil \frac{K}{2}\rceil+1},\ldots,n_K} \times \pi\left(n_1,n_2,\ldots,n_{\lceil \frac{K}{2}\rceil-1},N,n_{\lceil \frac{K}{2}\rceil+1},\ldots,n_K\right). \quad (62)$$

The handoff blocking probability experimented by users moving from cell $\lceil \frac{K}{2} \rceil$ to cell $\lceil \frac{K}{2} \rceil + 1$ serves to approximate P_H. Thus,

$$P_H = \left(\sum_{n_1,n_2,\ldots,n_{\lceil \frac{K}{2}\rceil},n_{\lceil \frac{K}{2}\rceil+2},\ldots,n_K} n_{\lceil \frac{K}{2}\rceil} \times \pi\left(n_1,n_2,\ldots,n_{\lceil \frac{K}{2}\rceil},N,n_{\lceil \frac{K}{2}\rceil+2},\ldots,n_K\right)\right) \times \left(\sum_{n_1,n_2,\ldots,n_K} n_{\lceil \frac{K}{2}\rceil}\pi(n_1,n_2,\ldots,n_K)\right)^{-1}. \quad (63)$$

Eqs. (59)–(63) form a set of simultaneous nonlinear equations which can be solved for system variables when parameters are given. For example given λ, μ, γ and N, the quantities P_B, P_H and $\pi(\vec{n})$ can be considered unknown. Beginning with an initial guess for P_B, P_H and $\pi(\vec{n})$, the equations may be solved numerically using the method of successive substitution. The forced termination probability P_T is approximated in the following way. The probability that an accepted call will fail at its first handoff is P_rP_H. The probability that an accepted call will fail at its second handoff is $P_rP_HP_r(1-P_H)$ (to attempt a second handoff, a call should have succeeded in its first handoff). The probability that an accepted call will attempt a kth handoff and then fail is $P_rP_HP_r^{k-1}(1-P_H)^{k-1}$. Thus,

$$P_T = P_rP_H\sum_{k=0}^{\infty} P_r^k(1-P_H)^k = \frac{P_rP_H}{1-P_r(1-P_H)}. \quad (64)$$

Therefore, once P_H is determined, the forced termination probability P_T is calculated using (64). To estimate the blocking probabilities, we have chosen to focus on cells located at the middle of the group since their statistical behaviors are expected to be the closest to the statistical behavior of the cells in the exact model.

3.2. Numerical results

Size of the group of isolated cells

It is clear that the new approximation may be of interest only if the number of isolated cells is small. Figure 5 shows that no more than one cell is needed to be isolated in order to obtain an approximation to the new call blocking probability. However, concerning the handoff blocking probability and the forced termination probability a group of two cells is needed (see figures 6 and 7). In these cases, there is a difference of about 10% between the results obtained by isolating a single cell and the results obtained by isolating a group of $K = 2$ cells. Figures 5, 6 and 7 all

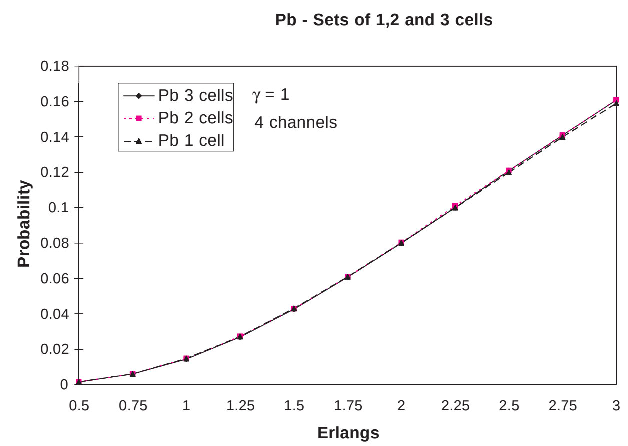

Figure 5. P_B vs load: Comparison between different numbers of isolated cells.

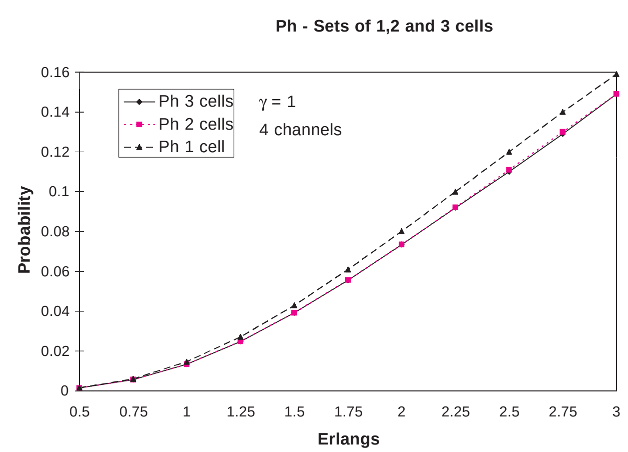

Figure 6. P_H vs load: Comparison between different numbers of isolated cells.

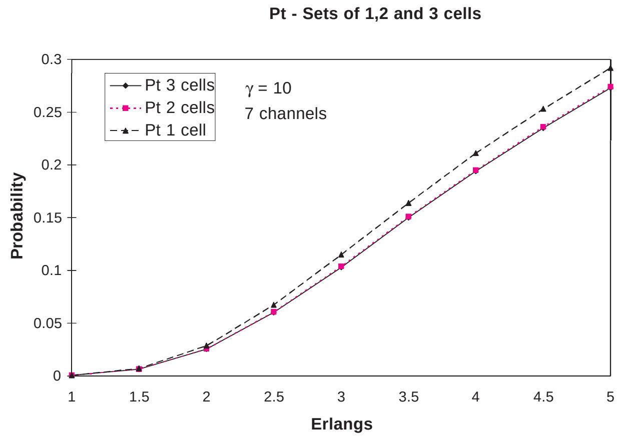

Figure 7. P_T vs load: Comparison between different numbers of isolated cells.

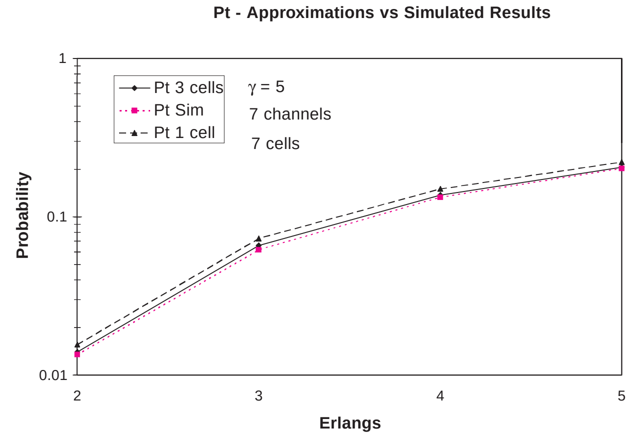

Figure 9. P_T vs load: Comparison between the approximated and the simulated results.

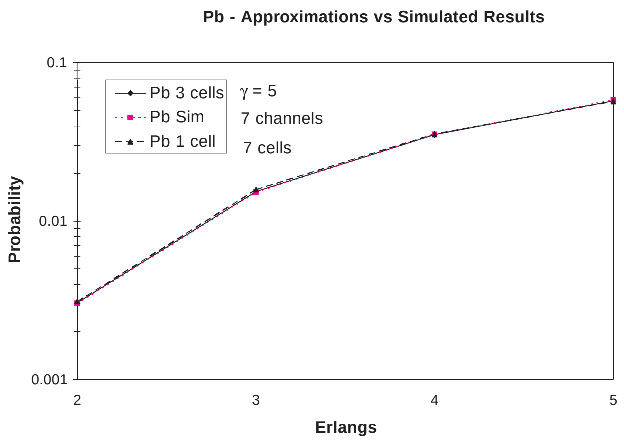

Figure 8. P_B vs load: Comparison between the approximated and the simulated results.

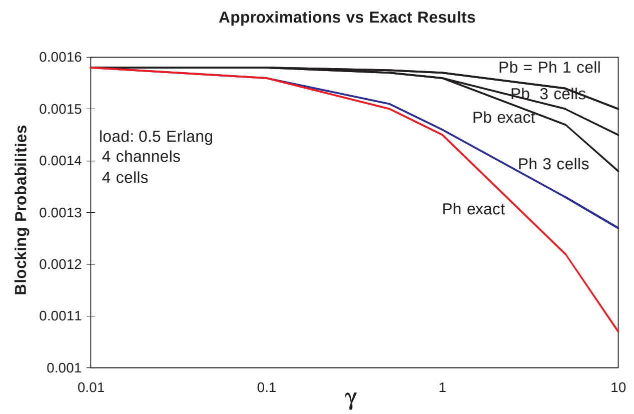

Figure 10. P_B and P_H vs γ: Comparison between the approximated and the exact results.

show that there is almost no difference between a group of two cells and a group of three cells. We come to the conclusion that choosing a value of $K = 2$ or $K = 3$ allows fast solution of the above equations. This also enables to distinguish between the new call blocking and the handoff blocking probabilities.

Validation of the approximation

The approximation needs to be validated by comparing approximated results with exact results. In figures 8 and 9 we consider a ring of $M = 7$ cells with $N = 7$ channels in each cell. The rate of handover is $\gamma = 5$. Figure 8 shows the new call blocking probability P_B versus the load in each cell for our approximation (3 cells isolated), for the approximation of [9] (1 cell isolated) and for results obtained by simulation. We see that both approximations are very accurate. However, as one can see from figure 9, our approximation to P_T is better than the approximation of [9]. We considered also the case of a small network of 4 cells with 4 channels. The new calls arrival rate λ is 0.5 and the rate of call completion is $\mu = 1$. The load $\rho = \lambda/\mu$ is thus 0.5 Erlang. The exact results were calculated from eqs. (1)–(4).

Figure 10 shows the blocking probabilities P_B and P_H versus γ for the two approximation approaches and for the exact results. The new approximation which distinguishes between the two kinds of blocking leads to a better accuracy than the approximation of [9], especially for the quantity P_H. One can see that our approximation is very good up to moderately high values of γ. From eq. (7), one obtains that the blocking probabilities in the very slow mobility regime are 0.00158. From figure 10, we see that as long as $\gamma/(\mu + \gamma) \ll 1$ (i.e., $\gamma < 0.1$), the blocking probabilities can be approximated by this value. We observe that as γ increases towards high values, both approximations become less accurate. However, our approximation is always closer to the accurate model than that of [9]. When γ tends to very high values, P_T tends to 1 and P_B tends to 0 for both approximations, while in the accurate model P_T tends to 9.5×10^{-2} and P_B tends to 3.7×10^{-4}. Our approximation is therefore validated except in the case of very small networks with very fast moving users.

4. Discussion and open problems

This paper has demonstrated that the usual assumptions made in the literature which do not differentiate between the new call blocking probability and the handoff blocking probability may be incorrect. As we have seen, the difference between the two kinds of blocking is particularly significant when the users move fast (or when the cells are very small), namely, $\gamma \gg \mu$ and $\gamma \gg \lambda$. Our numerical results show that if γ is larger than μ by at least three orders of magnitude, the blocking probabilities can be approximated by the expressions derived for the very fast mobility regime.

From eqs. (23) and (39), it may easily be shown that for fast users as the number of cells increases, the value of P_B decreases to very low values and the value of P_T is approaching 1. For example, if we consider a ring of 20 cells with 4 channels in each cell and a load of 0.5 Erlang in each cell, the value of P_T tends to 0.64 and the value of P_B tends to 4.04×10^{-6}, when γ tends to infinity. If we make the reasonable conjecture that the difference between P_B and P_H is an increasing monotic function of γ, the difference between them is bounded to 4.04×10^{-6}. Therefore, as was noted also in [15], the models considering handoff traffic as a Poisson process are reasonable when dealing with homogeneous traffic between a large number of cells. The new approximation approach that we have introduced in the previous section will always yield a better accuracy. Nevertheless, when considering networks with a small number of cells or networks with non-homogeneous traffic, it is preferable to use an exact model based on a multi-dimensional CTMC (as shown in section 2). An open problem is to find a sufficiently fine approximation approach which could also simplify this kind of multi-dimensional Markov chain.

In [10] where the case of 2 cells with one channel each was analyzed, it has been shown that for any $\gamma > 0$, $P_B > P_H$. We have shown in this paper that for any number of cells and for any number of channels $P_B > P_H$ when $\gamma \to \infty$. Another open problem is to prove that $P_B > P_H$ for any $\gamma > 0$, for any number of cells and for any number of channels

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Appendix A

Proof, by induction, of eq. (49).

1. For $k = 1$ we obtain from eq. (49) that $P_{D_1} = P_{D_1}$. Thus, eq. (49) is obviously fulfilled for $k = 1$. We determine now P_{D_2}. From eq. (45),

$$P_{D_2} = \left(\frac{\Lambda + \mu}{\Lambda}\right) P_{D_1}, \tag{65}$$

which also fulfills eq. (49) for $k = 2$.

2. Suppose that

$$P_{D_k} = \frac{P_{D_1}}{\Lambda^{k-1}} \sum_{i=0}^{k-1} \frac{(k-1)!}{(k-1-i)!} \Lambda^{k-1-i} \mu^i;$$
$$P_{D_{k+1}} = \frac{P_{D_1}}{\Lambda^k} \sum_{i=0}^{k} \frac{k!}{(k-i)!} \Lambda^{k-i} \mu^i. \tag{66}$$

We wish to show that

$$P_{D_{k+2}} = \frac{P_{D_1}}{\Lambda^{k+1}} \sum_{i=0}^{k+1} \frac{(k+1)!}{(k-i+1)!} \Lambda^{k-i+1} \mu^i.$$

From eq. (46) we have

$$P_{D_{k+2}} = \frac{P_{D_{k+1}}(\Lambda + (k+1)\mu) - k\mu P_{D_k}}{\Lambda}. \tag{67}$$

Inserting eq. (66) in eq. (67) we obtain

$$\begin{aligned}
P_{D_{K+2}} &= \frac{P_{D_1}}{\Lambda^{k+1}} \Bigg((\Lambda + (k+1)\mu) \sum_{i=0}^{k} \frac{k!}{(k-i)!} \Lambda^{k-i} \mu^i \\
&\qquad - k\mu\Lambda \sum_{i=0}^{k-1} \frac{(k-1)!}{(k-1-i)!} \Lambda^{k-1-i} \mu^i \Bigg) \\
&= \frac{P_{D_1}}{\Lambda^{k+1}} \Bigg(\sum_{i=0}^{k} \frac{k!}{(k-i)!} \Lambda^{k-i+1} \mu^i \\
&\qquad + \sum_{i=0}^{k} \frac{(k+1)!}{(k-i)!} \Lambda^{k-i} \mu^{i+1} \\
&\qquad - \sum_{i=0}^{k-1} \frac{k!}{(k-1-i)!} \Lambda^{k-i} \mu^{i+1} \Bigg) \\
&= \frac{P_{D_1}}{\Lambda^{k+1}} \Bigg(\Lambda^{k+1} + \sum_{i=1}^{k} \frac{k!}{(k-i)!} \Lambda^{k-i+1} \mu^i \\
&\qquad - \sum_{i=0}^{k-1} \frac{k!}{(k-1-i)!} \Lambda^{k-i} \mu^{i+1} \\
&\qquad + \sum_{i=0}^{k} \frac{(k+1)!}{(k-i)!} \Lambda^{k-i} \mu^{i+1} \Bigg) \\
&= \frac{P_{D_1}}{\Lambda^{k+1}} \sum_{i=0}^{k+1} \frac{(k+1)!}{(k-i+1)!} \Lambda^{k-i+1} \mu^i.
\end{aligned}$$

References

[1] G. Calhoun, *Digital Cellular Radio* (Artech House, 1988).

[2] CCITT Draft Recommendation E.771, Network grade of service parameters and target values for circuit-switching terrestrial mobile services, CCITT SWP II/3A, Geneva (February 1992).

[3] I. Cidon and M. Sidi, A multi-station packet-radio network, Performance Evaluation 8(1) (1988) 65–72; (see also *INFOCOM '84*, pp. 336–343).

[4] D. C. Cox, Wireless Personal Communications: What is it?, IEEE Personal Communications 2(2) (1995) 20–35.

[5] E. Del Re, R. Fantacci and G. Giambane, Handover and dynamic channel allocation techniques in mobile cellular networks, IEEE Transactions on Vehicular Technology 44(2) (1995) 229–237.

[6] G. Foschini, B. Gopinath and Z. Miljanic, Channel cost of mobility, IEEE Transactions on Vehicular Technology 42(4) (1993) 414–424.

[7] D. J. Goodman, Cellular packet communications, IEEE Trans. on Communications 38 (1990) 1272–1280.

[8] R. Guérin, Channel occupancy time distribution in a cellular radio system, IEEE Transactions on Vehicular Technology 36(3) (1987) 89–99.

[9] D. Hong and S. S. Rappaport, Traffic model and performance analysis for cellular mobile radio telephone systems with prioritized and nonprioritized handoff procedures, IEEE Transactions on Vehicular Technology 35(3) (1986) 77–92.

[10] Y. Hou, Call handover in mobile cellular networks, Preprint.

[11] H. Jiang and S. S. Rappaport, CBWL: A new channel assignment and sharing method for cellular communications systems, in: *Proc. of 43rd Vehicular Technology Conference* (May 1993) pp. 189–193.

[12] W. Lee, *Mobile Cellular Telecommunications Systems* (Mcgraw-Hill, New York, 1989).

[13] E. H. Lipper and M. P. Rumsewicz, Teletraffic considerations for widespread deployment of PCS, IEEE Network 8(5) (1994) 40–49.

[14] D. McMillan, Traffic modeling and analysis for cellular mobile networks, in: *Proc. of 13th Int. Teletraffic Congress* (1991) pp. 627–632.

[15] M. Naghshineh and A. S. Acampora, Design and control of microcellular networks with QOS provisioning for real-time traffic, in *ICUPC '94* (October 1994).

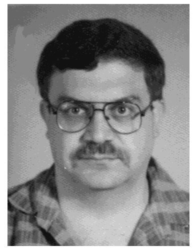
[16] J. E. Padgett, C. G. Günther and T. Hattori, Overview of wireless personal communications, IEEE Communications Magazine 33(1) (1995) 28–41.

[17] D. L. Schilling, Wireless communications going into the 21st century, IEEE Transactions on Vehicular Technology 43(3) (1994) 645–651.

[18] K. Sohraby, Blocking and forced termination in pico-cellular wireless networks: An asymptotic analysis, in: *Proc. of 9th Annual Workshop on Computer Communications* (1994).

[19] R. Steele, The cellular environment of lightweight hand held portables, IEEE Communications Magazine 10 (1989) 20–29.

[20] S. Tekinay and B. Jabbari, Handover and channel assignment in mobile cellular networks, IEEE Communications Magazine 30(11) (1991) 42–46.



Moshe Sidi received the B.Sc., M.Sc. and the D.Sc. degrees from the Technion – Israel Institute of Technology, Haifa, Israel, in 1975, 1979 and 1982, respectively, all in electrical engineering. In 1982 he joined the faculty of Electrical Engineering Department at the Technion. During the academic year 1983–1984 he was a Post-Doctoral Associate at the Electrical Engineering and Computer Science Department at the Massachusetts Institute of Technology, Cambridge, MA. During 1986–1987 he was a visiting scientist at IBM, Thomas J. Watson Research Center, Yorktown Heights, NY. He coauthors the book "Multiple Access Protocols: Performance and Analysis", Springer-Verlag, 1990.

He served as the Editor for Communication Networks in the *IEEE Transactions on Communications* from 1989 until 1993, and as the Associate Editor for Communication Networks and Computer Networks in the *IEEE Transactions on Information Theory* from 1991 until 1994. Currently he serves as an Editor in the *IEEE/ACM Transactions on Networking* and as an Editor in the *Wireless Journal*. His research interests are in wireless networks and multiple access protocols, traffic characterization and guaranteed grade of service in high-speed networks, queueing modeling and performance evaluation of computer communication networks.

E-mail: moshe@ee.technion.ac.il

David Starobinski was born in 1971 in Geneva, Switzerland. He received the B.Sc. (Cum Laude) and the M.Sc degrees from the Technion – Israel Institute of Technology, Haifa, Israel, in 1993 and 1996, respectively, all in electrical engineering. Since 1993, he has been a research assistant and a teaching assistant at the Electrical Engineering Department at the Technion. In 1991, he was granted the Howard and Margaret Lane Scholarship Award. In 1996, he was granted the Gutwirth Scholarship award. He is now pursuing his studies towards a D.Sc. degree in electrical engineering, at the Technion.

His research interests are in performance evaluation of computer communication networks and wireless networks, and in simulations modelling.

E-mail: staro@tx.technion.ac.il